Bayesian ESS-M

Raymond Nelson

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Bayes – Vocabulary Primer

- Bayesian inference
- Bayes Theorem
- Probability (Bayesian probability)
- Prior probability (prior probability distribution or *a priori*)
- Likelihood function
- Posterior probability (*a posteriori*)
- Odds
- Bayes Factor
- Credible interval
- Naive-Bayes
- Objective Bayesian Analysis
- Subjective Bayesian Analysis

Bayesian inference

- Statistical inference is the process of using sampling and data analysis to estimate a quantity of interest (Jeffreys, 1961; Savage, 1954) – referred to as an *unknown parameter*.
- Inference is necessary when the parameter of interest cannot be subject to deterministic observation or physical measurement.
- Bayesian inference (Box & Tiao, 1973; Jaynes, 1986, 2003) involves the use of Bayes' theorem to estimate the unknown parameter of interest.

Bayes Theorem

- A theorem is a mathematical expression that has been subjected to extensive mathematical proof.
- Bayes’s theorem (Stigler, 1982; 1983) – also referred to as Bayes’ rule and Bayes’ law – is based on the work on Thomas Bayes (Bayes & Price, 1763) and Simon Pierre Laplace (1774/1986; 1812).
- Bayes theorem involves the use of new information to improve the confidence or reduce the uncertainty about a conclusion associated with some prior existing information.

Probability (Bayesian probability)

- Bayesian probability refers to the degree of belief one may hold in some knowledge or conclusion under uncertain circumstances (de Finetti, 2017; Jaynes, 2003).
  - Can also to an estimate of the reasonable expectation or likelihood for single trial (Cox, 1946).
- Frequentist probability refers to the frequency of observed events with an assumption that the circumstances can be subject to indefinite repetition.
  - Frequentist probabilities require phenomena that are both observable and repeatable.
  - Bayesian probabilities can be used with a wider range of observable and un-observable phenomena.

Prior probability
(prior probability distribution or *a priori*)

- The prior probability, sometimes referred as *a priori* or more simply *prior*, represents what is known about the likelihood of different possible outcomes before a scientific test or experiment is conducted (Berger, 1985; Rubin et al., 2003).
- The prior probability distribution can be based on objective or empirical information such as a base-rate or incidence rate.
  - For example: if exactly four persons had access and opportunity to commit a crime then the prior probability is not less than .25, or 1 in 4 chances.
- The optimal prior probability will often be 1 in 2 whenever little information is available – when there are 2 possible categorical conclusions.
- Bayesian outcomes can also be evaluated for a range of different possible prior probabilities.

Likelihood function

- A likelihood function (Jaynes, 2003; Rohde, 2014) is a device for obtaining a statistical or likelihood value associated with some data.
- A likelihood function can be thought of as a function of the input parameters that determine some probability distribution or statistical model.
  - Normal distribution
    - Mean
    - Standard deviation

Posterior probability
(*a posteriori*)

- The posterior probability tells us the revised probability or likelihood associated with a test result or conclusion after the available data are considered (Bernardo & Smith, 1994; Lee, 2004).
- The posterior is the combination, using Bayes' theorem, of the *prior*, and *likelihood function*, and *data* from a test or experiment.

Odds

- Odds are a convenient and intuitive way of discussion probabilistic information in a manner that is easily expressed in prose (Fulton, et al., 2012; Gelman et al., 2003).
  - For example: the odds obtaining a “head” when tossing a fair coin can be described as 1 to 1 or 1 in 2.
- We can calculate the odds for any probability or proportion
  - Odds = p / (1 – p)
- Also, if we know the odds we can calculate the proportion or probability
  - p = odds / (1 + odds)
- Odds may provide clearer and more intuitively useful information for some people
  - Odds measure chances of occurrence vs non-occurrence
  - Probability measures chance against a whole

Bayes Factor

- A Bayes factor (Berger, 2006a; Morey & Rouder, 2011; Rouder et al., 2009) *quantifies the strength of evidence*, from a scientific test or experiment, for one conclusion over another.
- Can also be thought of as the value for which we would multiply the prior odds to obtain the posterior odds.
- Bayes factor will be equal to the posterior probability whenever the prior odds are 1 to 1 but will differ from the posterior probability when the prior probability distribution is unequal.
- Bayes factors are similar to likelihood ratios and can provide an alternative to frequentist hypothesis testing using Bayesian inference (Goodman, 1999).

Credible interval

- A credible interval is the Bayesian analog for a confidence interval in frequentist statistics (Edwards, Lindman & Savage, 1963; Jaynes, 1976; Lee, 2004).
- Tells us the range of variability (i.e. how sure we are) that we can reasonably be about analytic result or conclusion.
- Bayesian inference regards the data as a fixed quantity of available information with which to calculate an interval that can be interpreted as indicative of the probability that the unknown parameter of interest exists within.
  - Frequentist inference views the available data as a random variable that is subject to expected variation upon replication
  - Frequentist confidence interval tells us the proportion of replications that will include an unknown parameter of interest

Naive-Bayes

- Naive-Bayes is a widely used application of Bayes' Theorem to statistical decision making, machine-learning and artificial intelligence (Hand & Yu, 2001; Russel & Norvig, 2009).
- In this case "naive" refers to the use of strong assumptions that the different sources of data are *independent* and *contribute equally* to the outcome (Domingos & Pazzani, 1997; Pazzani, 1996).
- Naive-Bayes algorithms are advantageous in that they are simpler to understand, rapid and easy to develop, and often perform well compared to more complex classifiers.

Objective Bayesian Analysis

- Objective-Bayesian Analysis (Berger, 2006b; Chen et al., 2010) refers to the use of Bayes' theorem with objective (non-subjective) prior information.
- Objectivity is an ideal of scientific inquiry and scientific testing.
- Completely objective information is often not available
  - Some have questioned whether the ideal of complete objectivity is an illusion (Feinberg, 2006).

Subjective Bayesian Analysis

- Subjective Bayesian Analysis (Duda, Hart & Nillson, 1981; Goldstein, 2006) refers to the use of Bayes' theorem with subjective (non-objective) prior information.
- Many important practical problems begin with prior information that is incomplete and subjective or reliant on interpretation.
- Subjective-Bayes methods are a framework for using data from a scientific test or experiment to obtain posterior probability estimates that have reduced error and uncertainty compared to the subjective prior information.

Bayes' Theorem

Bayes Theorem

$$P(A \mid B) = \frac{P(B \mid A)}{P(B)} \; P(A)$$

Bayes Theorem
In the form of a hypothesis test

$$P(H \mid E) = \frac{P(E \mid H)}{P(E)} P(H)$$

Bayes Theorem

$$P(H \mid E) = \frac{P(E \mid H)}{P(E)} P(H)$$

H = Hypothesis

E = Evidence (data)

P(E | H) = Test sensitivity rate (true-positive rate)

P(E) = True-positive rate + False-positive rate

P(H) = Prior probability (base rate)

P(H | E) = Posterior probability

Bayes' Theorem – rearranged

$$P(H \mid E) = \frac{P(E \mid H)}{P(E \mid H) + P(E \mid 1\text{-}H)} * P(H)$$

- H = Hypothesis
- E = Evidence (data)
- P(E | H) = Likelihood (sensitivity or true-positive rate)
- P(H) = Prior probability (base rate)
- P(H | E) = Posterior probability
- P(E | 1-H) = Likelihood compliment (false-hit rate)

Bayes’ Theorem – more rearrangement

$$P(H \mid E) = \frac{P(E \mid H) * P(H)}{P(E \mid H) * P(H) + P(E \mid 1\text{-}H) * (1\text{-}P(H))}$$

- P(H | E) = Posterior probability
- P(E | H) = Likelihood statistic
  - Multinomial likelihood
  - Observed score or lower
- P(H) = Prior probability (base rate)
- P(E | 1-H) = Likelihood compliment
- (1-P(H)) = Prior compliment (1 – base rate)

Plugging in the concepts

$$\text{posterior} = \frac{\text{likelihood} * \text{prior}}{\text{likelihood} * \text{prior} + \text{likelihood compliment} * \text{prior compliment}}$$

Let’s make some numbers and try it...

- Test score = +5

ESS-M Likelihood Function (3RQs)

score	ways	*pmf*	*cdf*	cdfContCor	odds	oddsLL05
-22	360	.0009*	.0025	.0021	483	17.34
-21	370	.0013	.0038	.0031	317.7	16.38
-20	381	.0018	.0056	.0047	212.8	15.18
-18	400	.0035	.0115	.0098	100.6	13.93
-17	408	.0047	.0162	.0139	70.88	12.03
-16	417	.0062	.0223	.0193	50.72	11.12
-15	424	.0080	.0301	.0264	36.84	9.86
-14	432	.0102	.0402	.0355	27.14	8.48
-13	438	.0128	.0526	.0471	20.25	7.15
-12	445	.0157	.0680	.0613	15.31	6.13
-11	450	.0190	.0864	.0787	11.7	5.15
-10	456	.0226	.1081	.0996	9.04	4.27
-9	460	.0264	.1335	.1242	7.05	3.57
-8	465	.0304	.1624	.1526	5.55	2.99
-7	468	.0343	.1950	.1850	4.4	2.48
-6	472	.0382	.2310	.2213	3.52	2.05
-5	474	.0418	.2703	.2613	2.83	1.69
-4	477	.0449	.3125	.3046	2.28	1.4
-3	478	.0476	.3571	.3508	1.85	1.15
-2	480	.0495	.4036	.3992	1.51	0.95
-1	480	.0508	.4515	.4492	1.23	0.77
0	481	.0512	.5000	.5000	1	0.63
1	480	.0508	.5485	.5508	1.23	0.77
2	480	.0495	.5964	.6008	1.51	0.95
3	478	.0476	.6429	.6492	1.85	1.15
4	477	.0449	.6875	.6954	2.28	1.4
5	474	.0418	.7297	.7387	2.83	1.69
6	472	.0382	.7690	.7787	3.52	2.05
7	468	.0343	.8050	.8150	4.4	2.48

Let’s make some numbers and try it...

- Test score = +5
- Likelihood (cdfContCor) = .7387 (2.83 to 1)

Let’s make some numbers and try it...

- Test score = +5
- Likelihood (cdfContCor) = .7387 (2.83 to 1)
- Prior = .5 (1 to 1)
- Prior compliment = .5 (1-.5)
- Likelihood compliment = .2613 (1-.7387)

Now plugging in the numbers...

$$\text{posterior} = \frac{.7387 * .5}{.7387 * .5 + .2613 * (1-.5)}$$

Now plugging in the numbers...

$$.7387 = \frac{.7387 * .5}{.7387 * .5 + .2613 * (1-.5)}$$

Why bother with the math?

- Because sometimes the prior is not 1 to 1
- Now that we have the math we can use any prior we want

More about priors

Bayesian Analysis

- Starts with a prior probability (prior odds) for a hypothesis
  - Prior odds of deception
  - Prior odds of truth
- Uses Bayes theorem
- Ends with a posterior probability (posterior odds)
  - Odds of deception
  - Odds of truth

Bayes – Where does the prior come from?

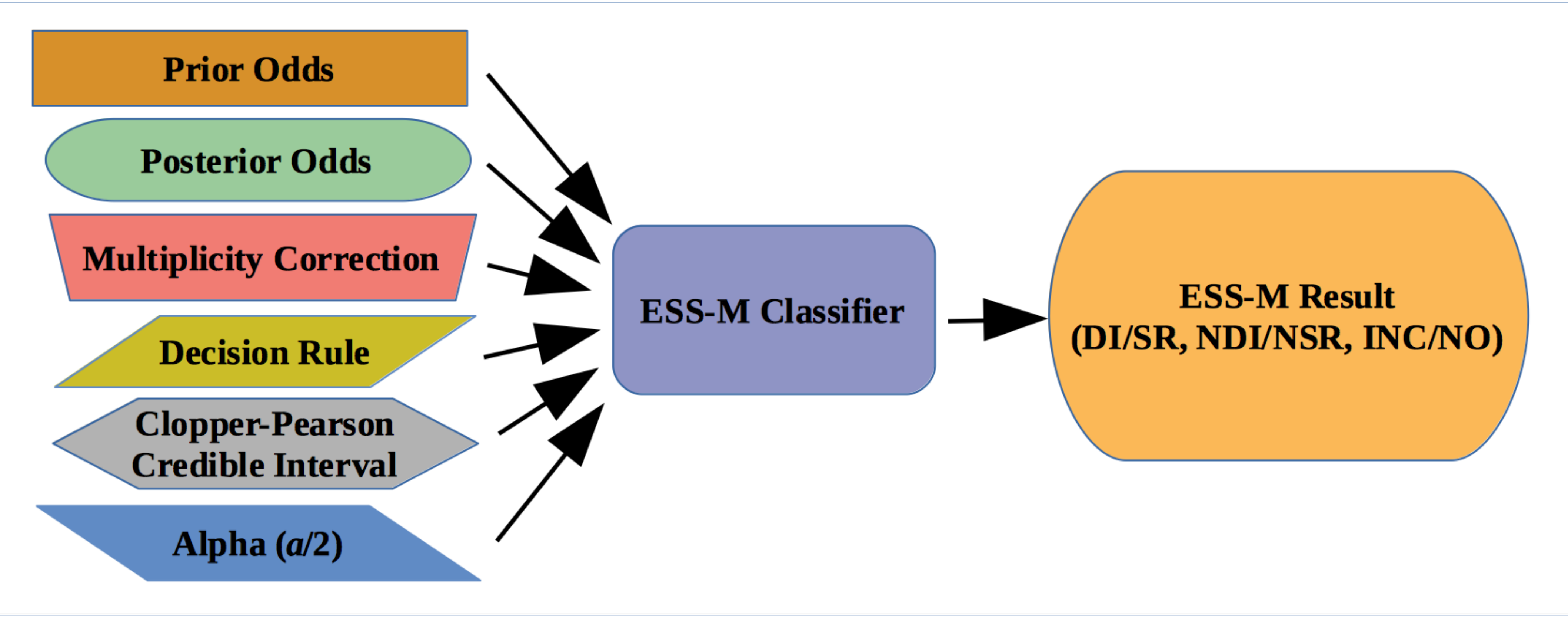
- Objective information
  - Base rate or incidence rate for a known population
  - Number of suspects
  - Previous analytic result
- Weak information
  - Some reason for testing
  - Possibly innocent
  - Information is insufficient to conclude either deception or truth
  - Number of different possible results (DI/SR or NDI/NSR)

Bayes – Where does the prior come from?

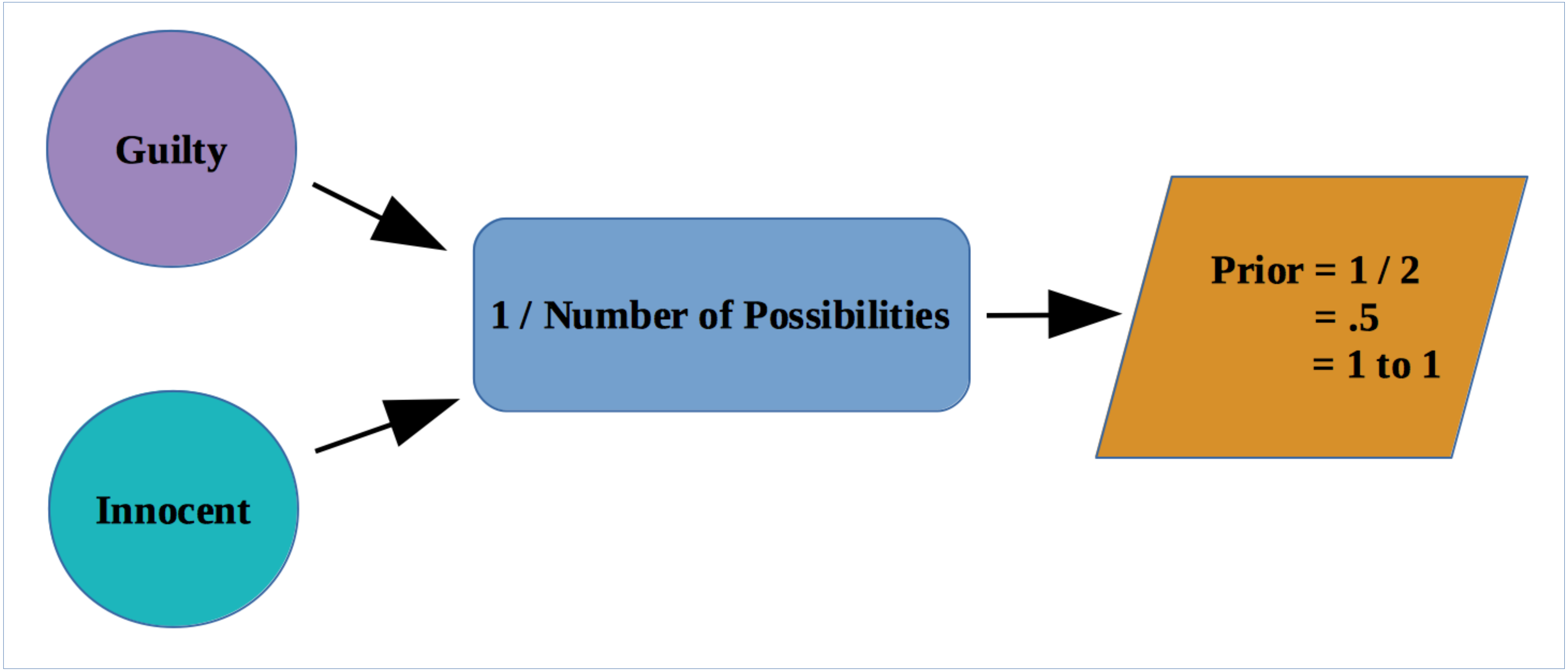
- The prior can be estimated from the number of different possible conclusions when there is no information or the information is very weak or insufficient to support any conclusion
  - Deceptive
  - Truthful
- Prior = 1 to 1 (.5)

Bayesian ESS-M Classifier

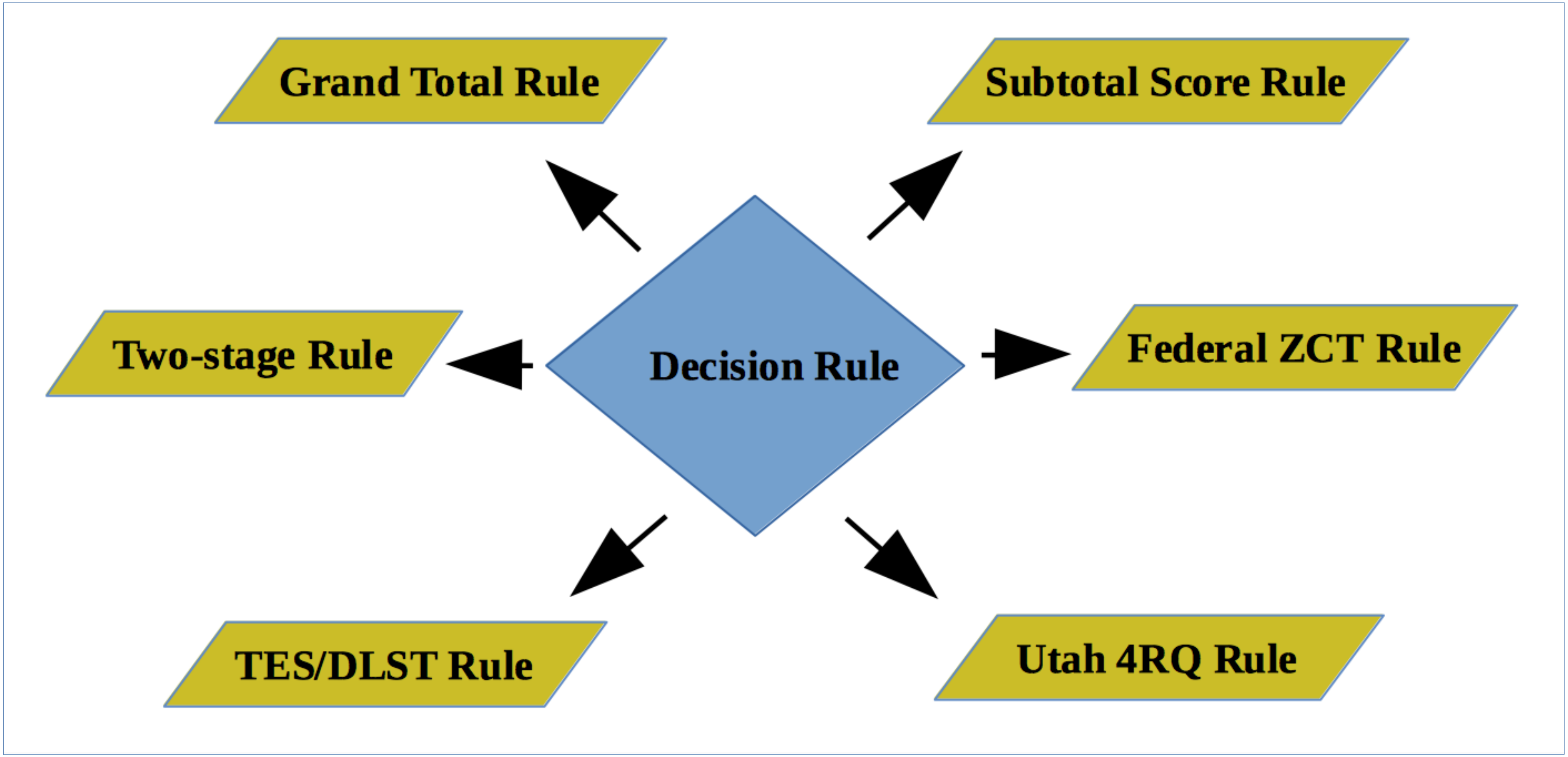
ESS-M Classifier



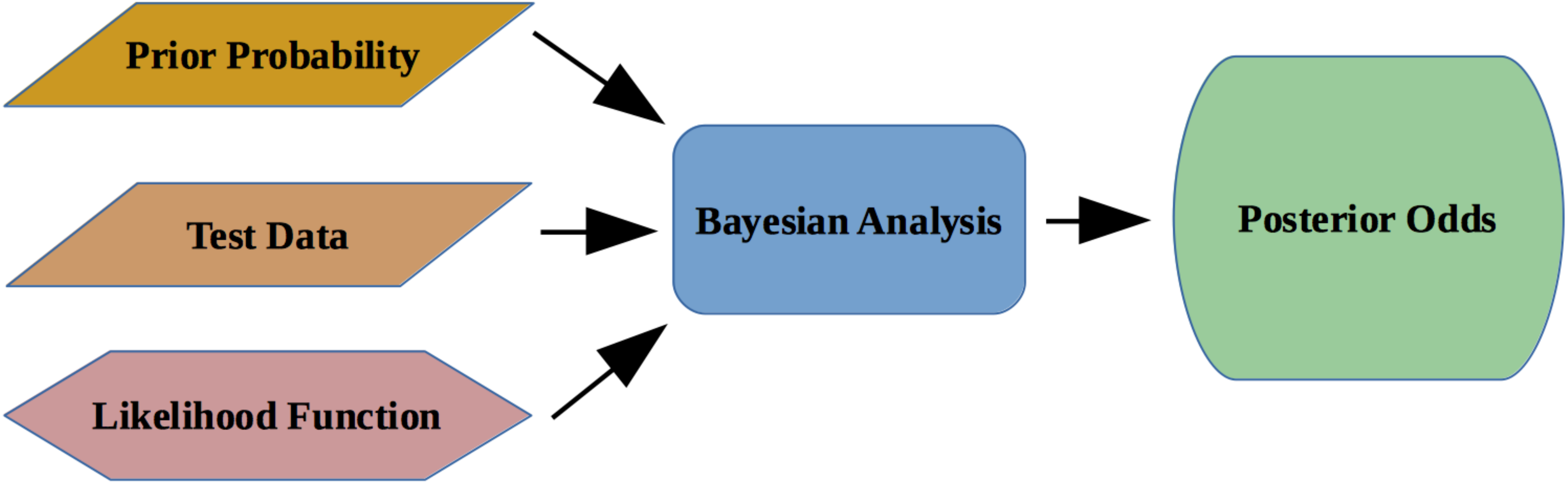
Prior Odds



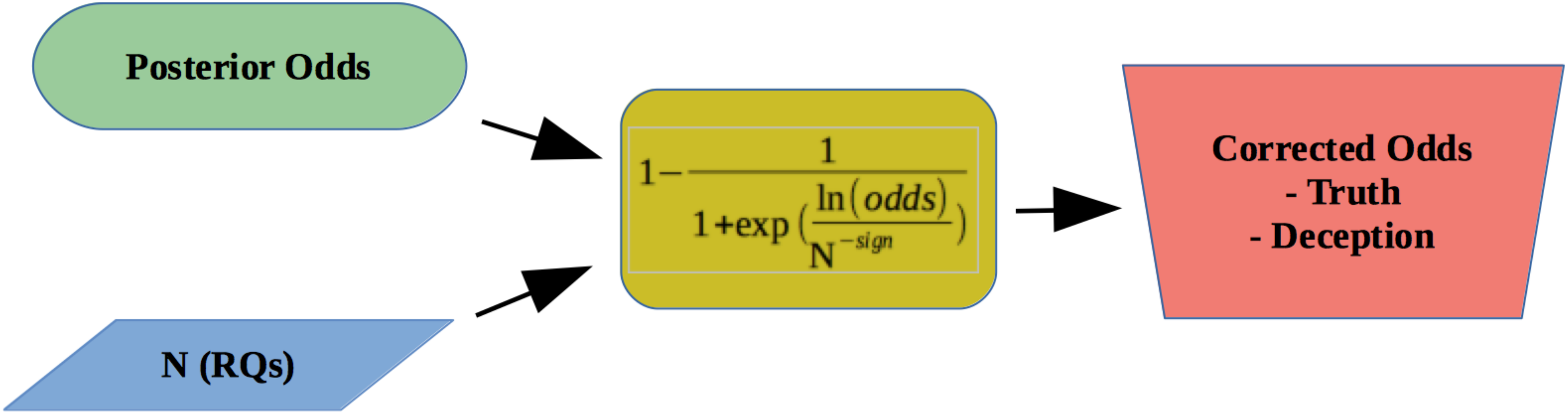
Decision Rule



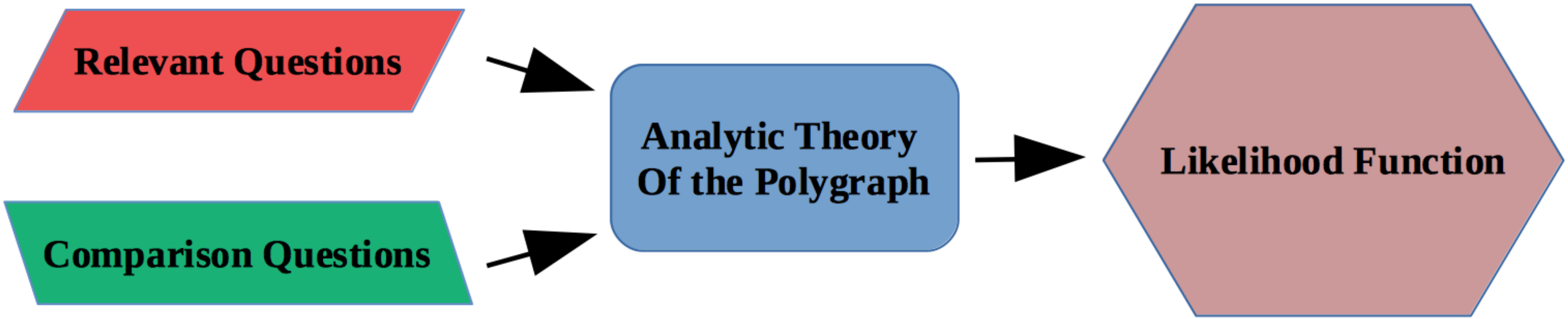
Bayesian Analysis



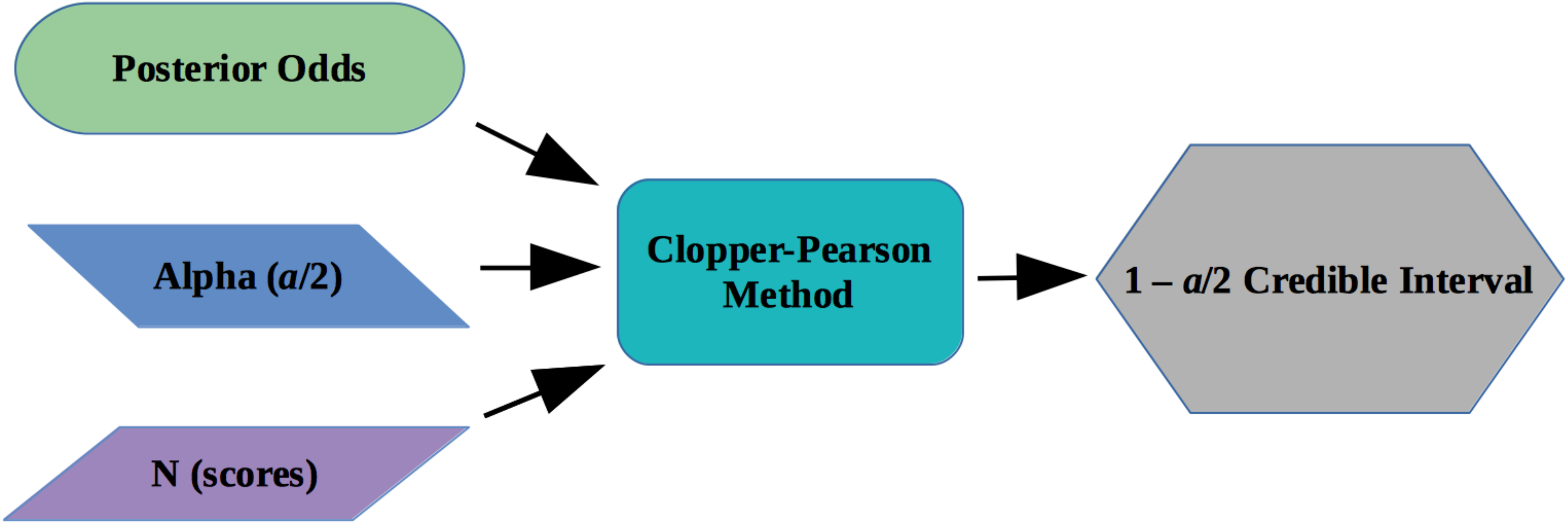
Multiplicity Correction for Odds



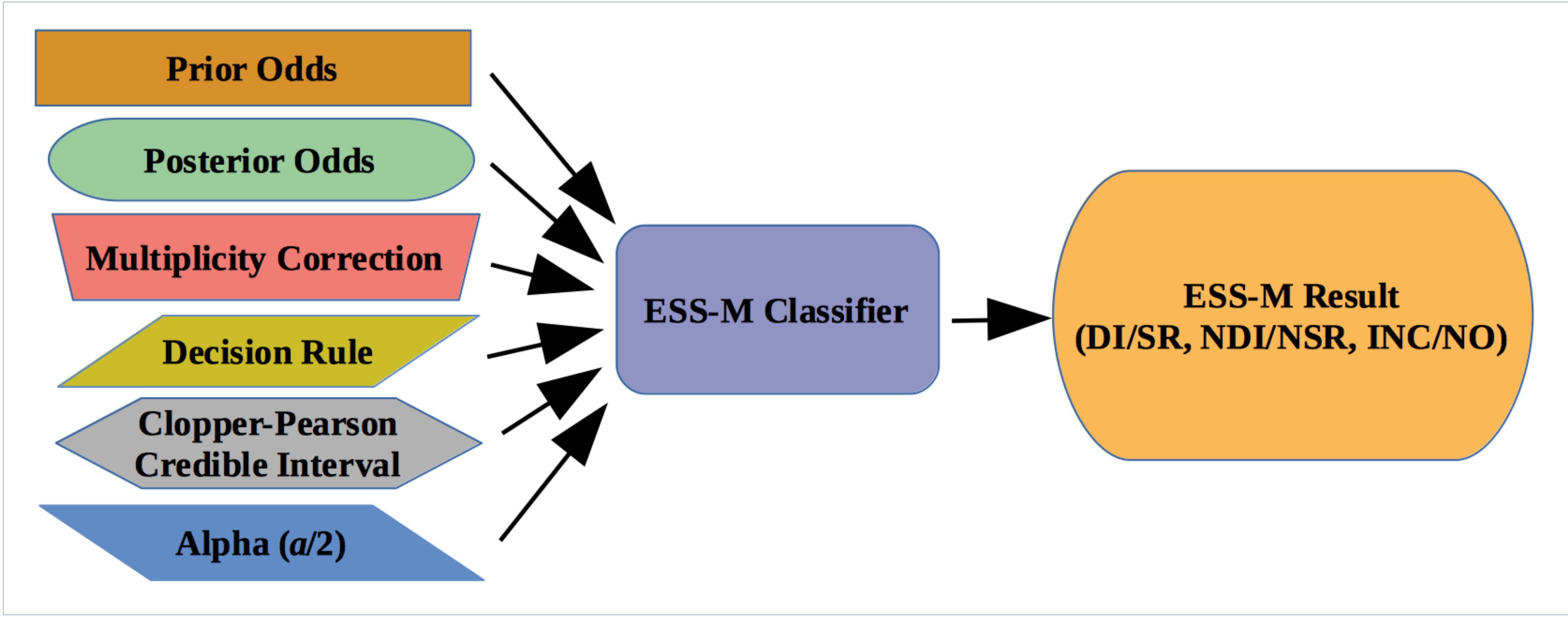
Multinomial Likelihood Function



Credible Interval



ESS-M Classifier



Bayesian ESS-M Classifier

Bayesian ESS-M Classifier

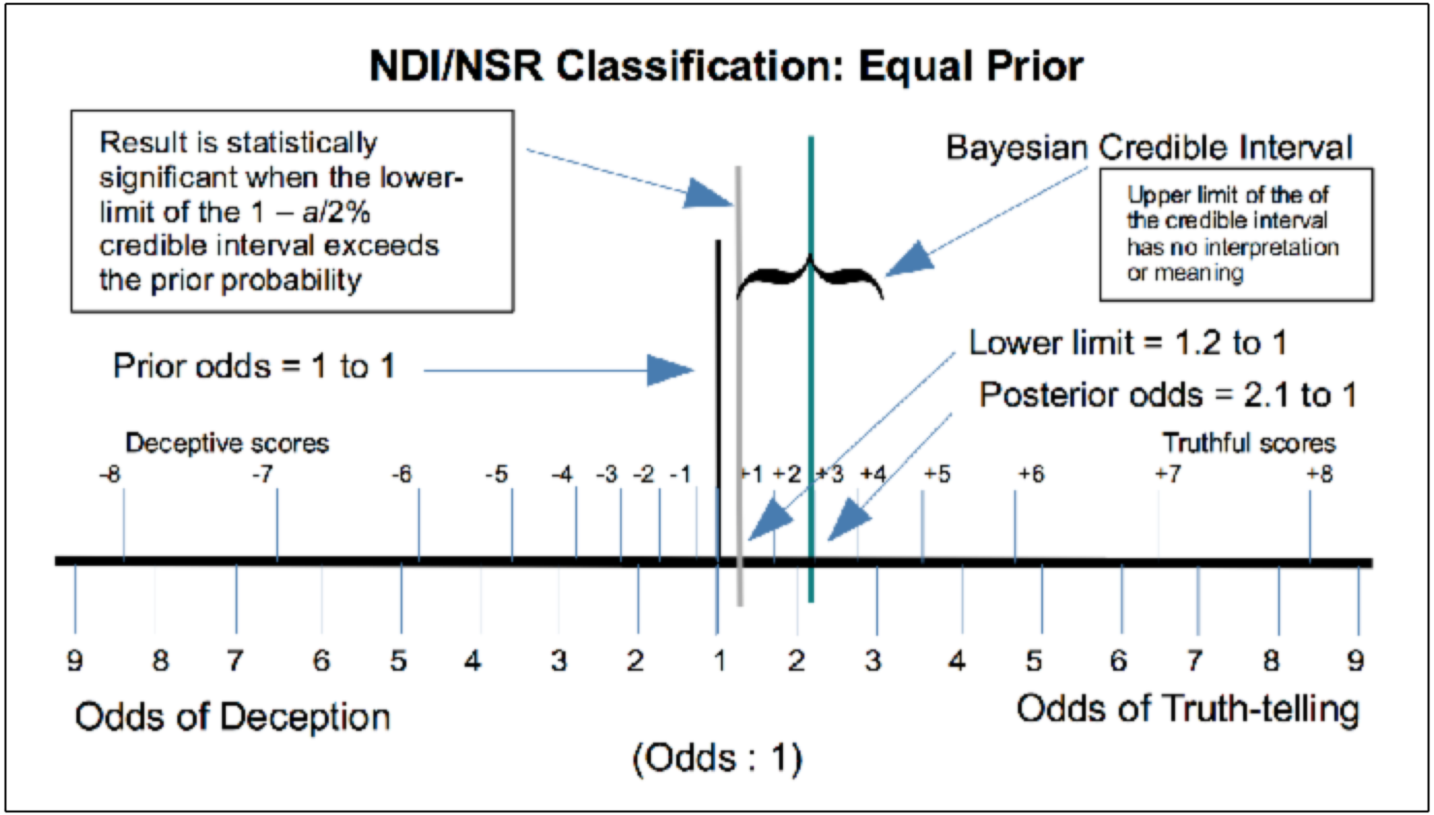
- Combination of
  - Prior odds
    - Prior information, or
    - Number of possibilities
  - Posterior odds
    - Likelihood Function
      - Analytic theory of the polygraph test
    - Decision rule
    - Multiplicity correction
  - Clopper-Pearson interval
    - Posterior odds
    - Alpha
    - Number of RQs x repetitions
  - Alpha level

Bayesian ESS-M Classifier

- A test result is statistically significant when the lower-limit of the credible interval has exceeded the prior probability

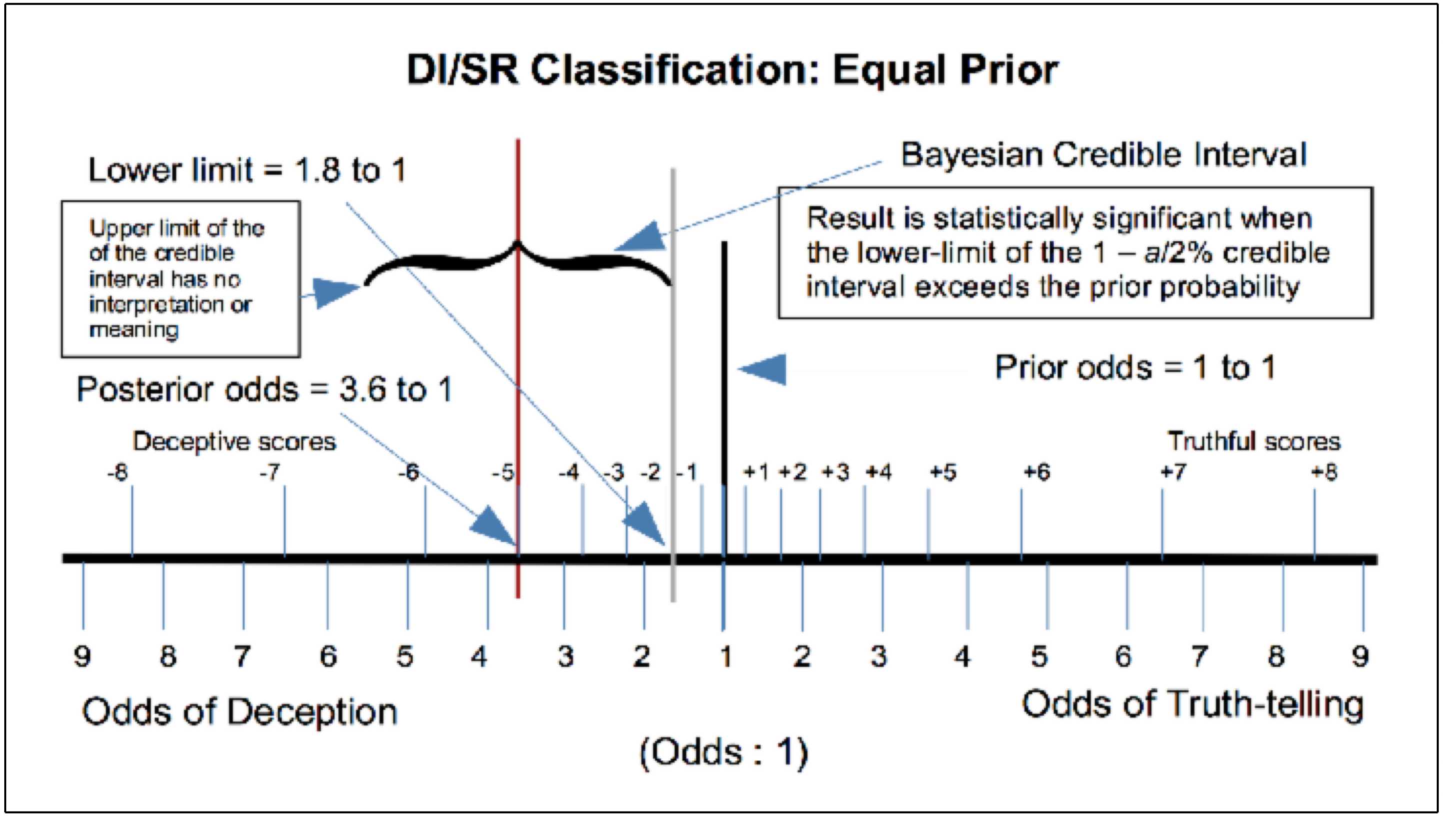
NDI/NSR (Equal Prior)

Figure 2. NDI/NSR results with prior = 1 to 1 (.5) and alpha = .05.



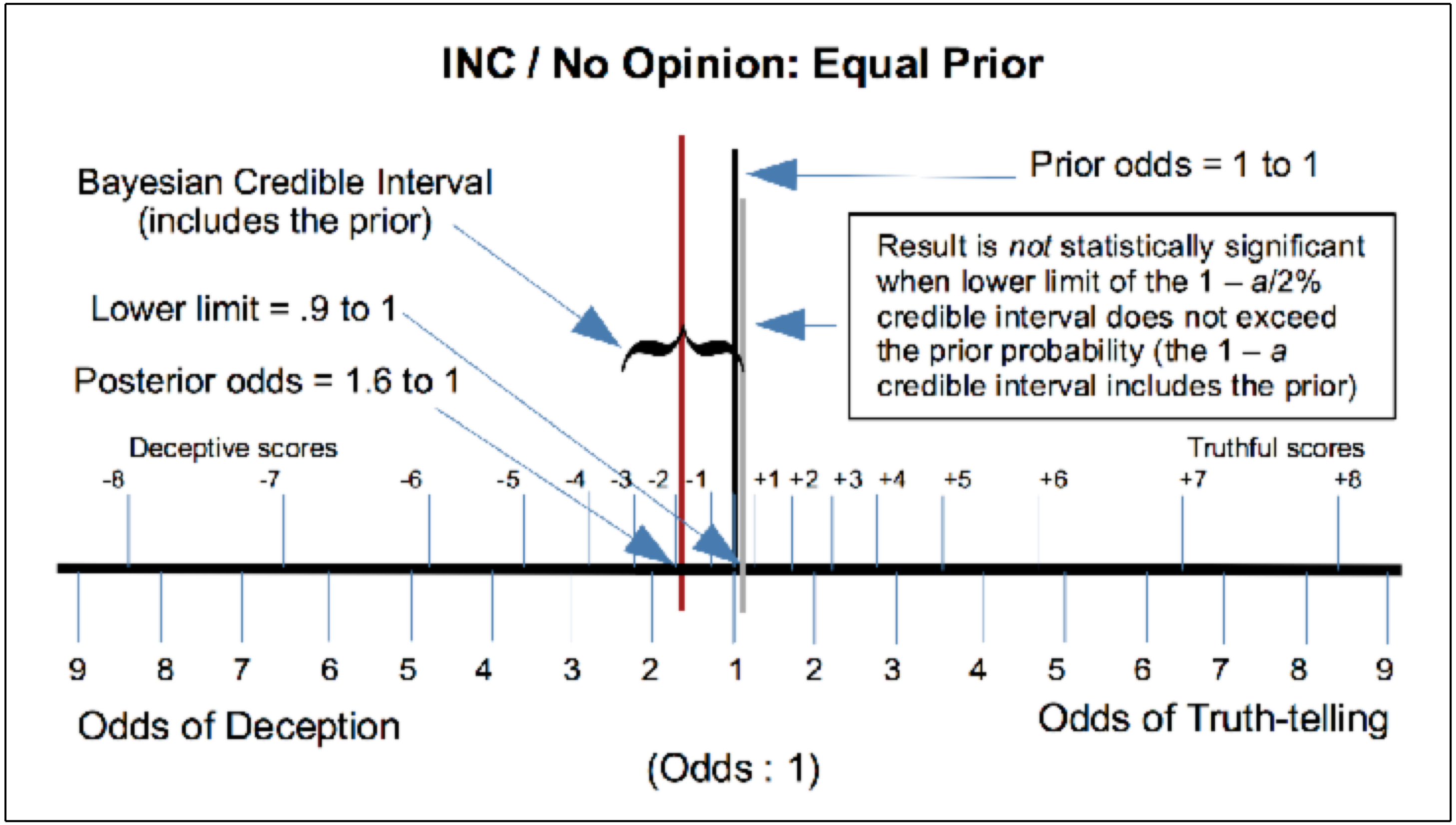
DI/SR (Equal Prior)

Figure 3. DI/SR results with prior = 1 to 1 (.5) and alpha = .05.



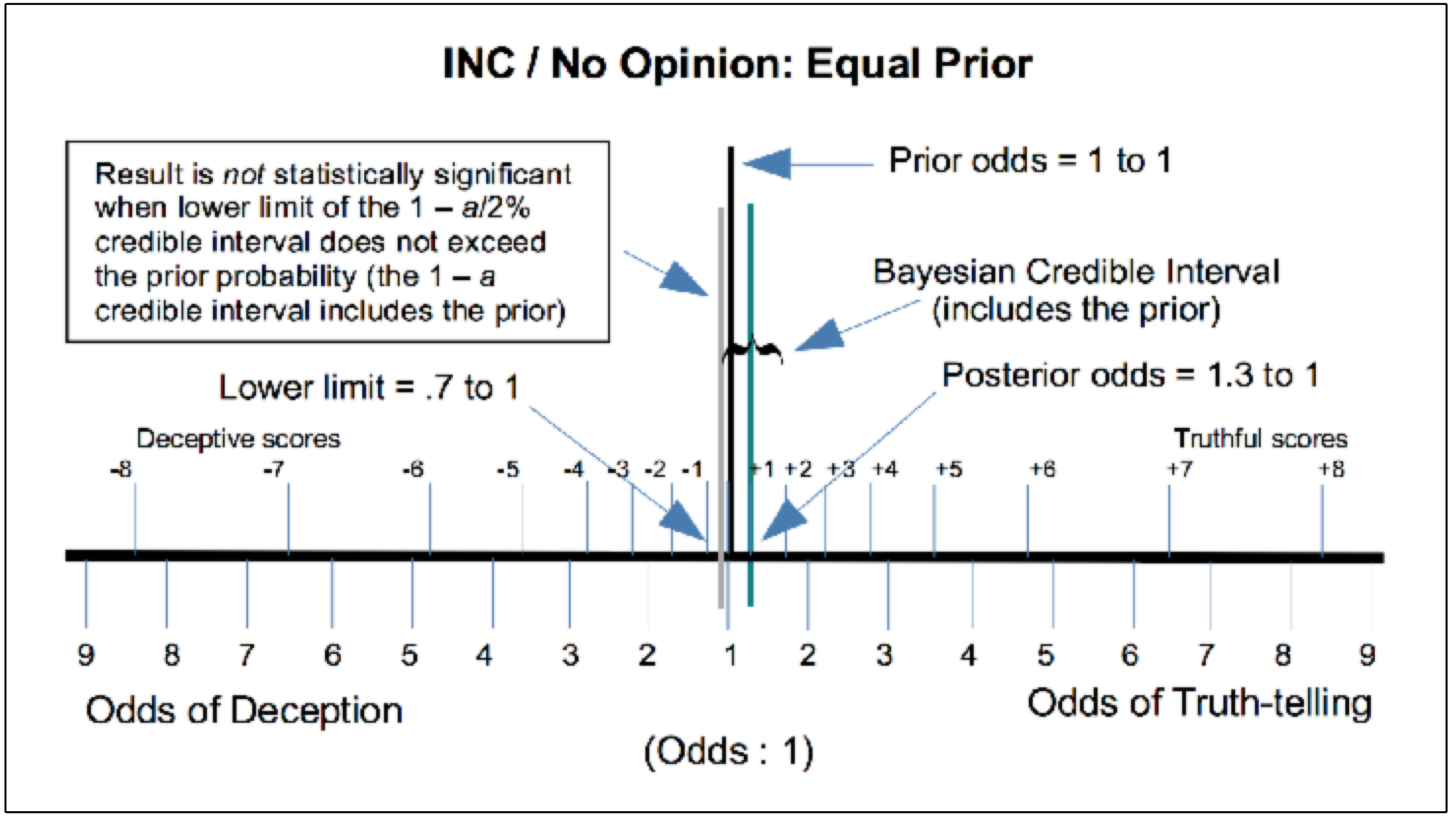
INC/NO (Equal Prior)

Figure 4. Inconclusive result with prior = 1 to 1 (.5) and alpha = .05.



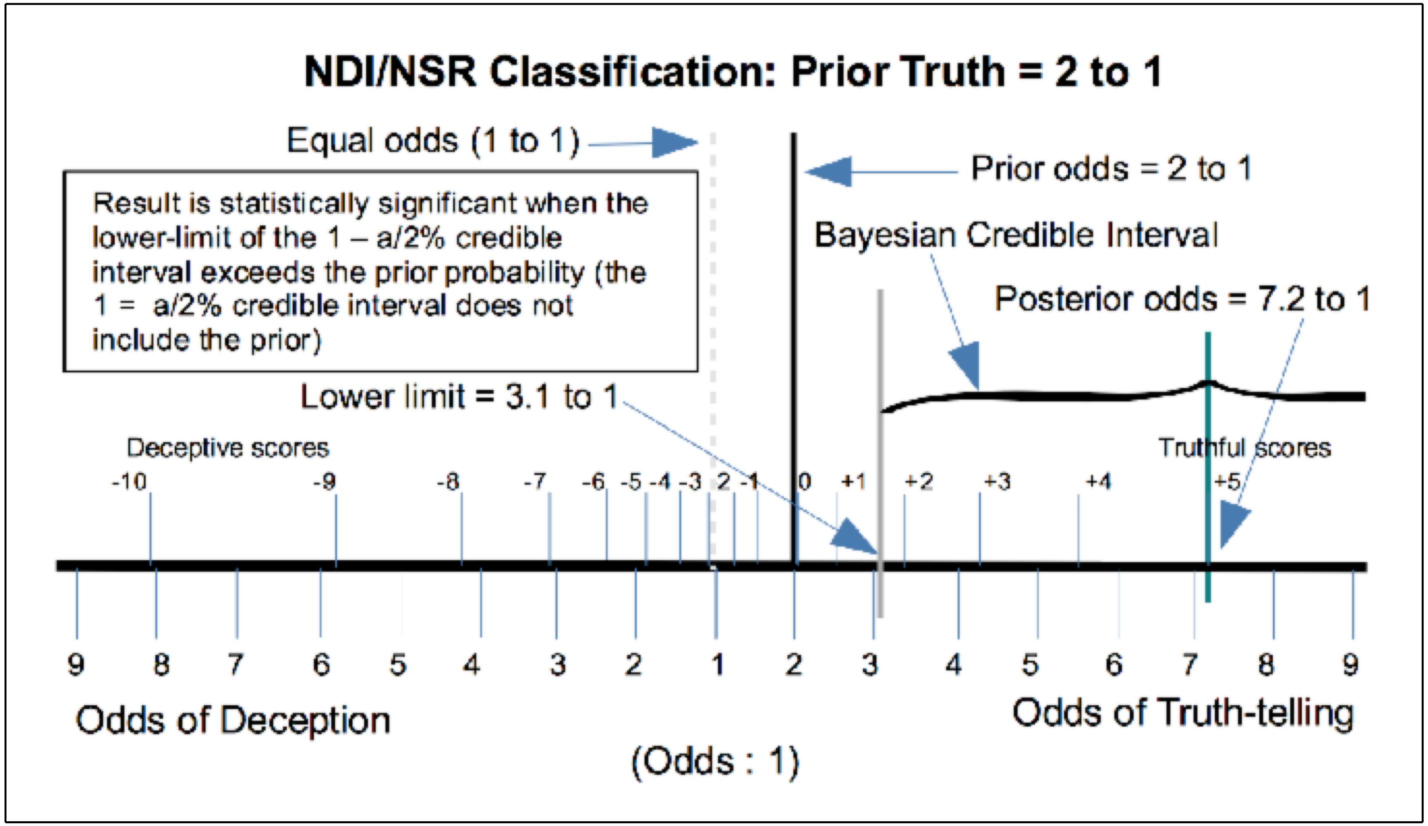
INC/NO (Equal Prior)

Figure 5. Inconclusive result with prior = 1 to 1 (.5) and alpha = .05.



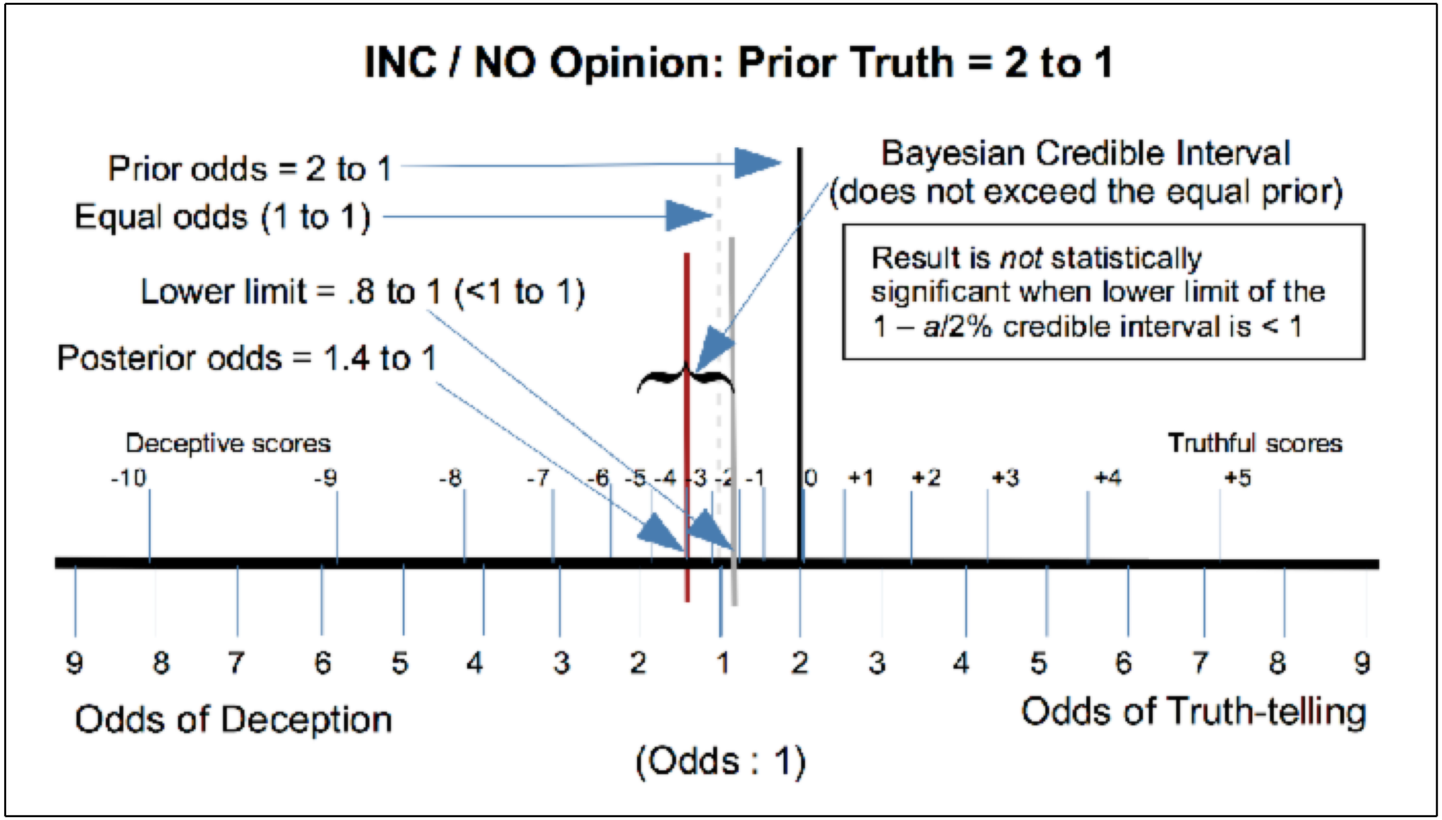
NDI/NSR (Prior = 2 to 1)

Figure 6. Truthful test result with prior odds of truth = 2 to 1 and alpha = .05.



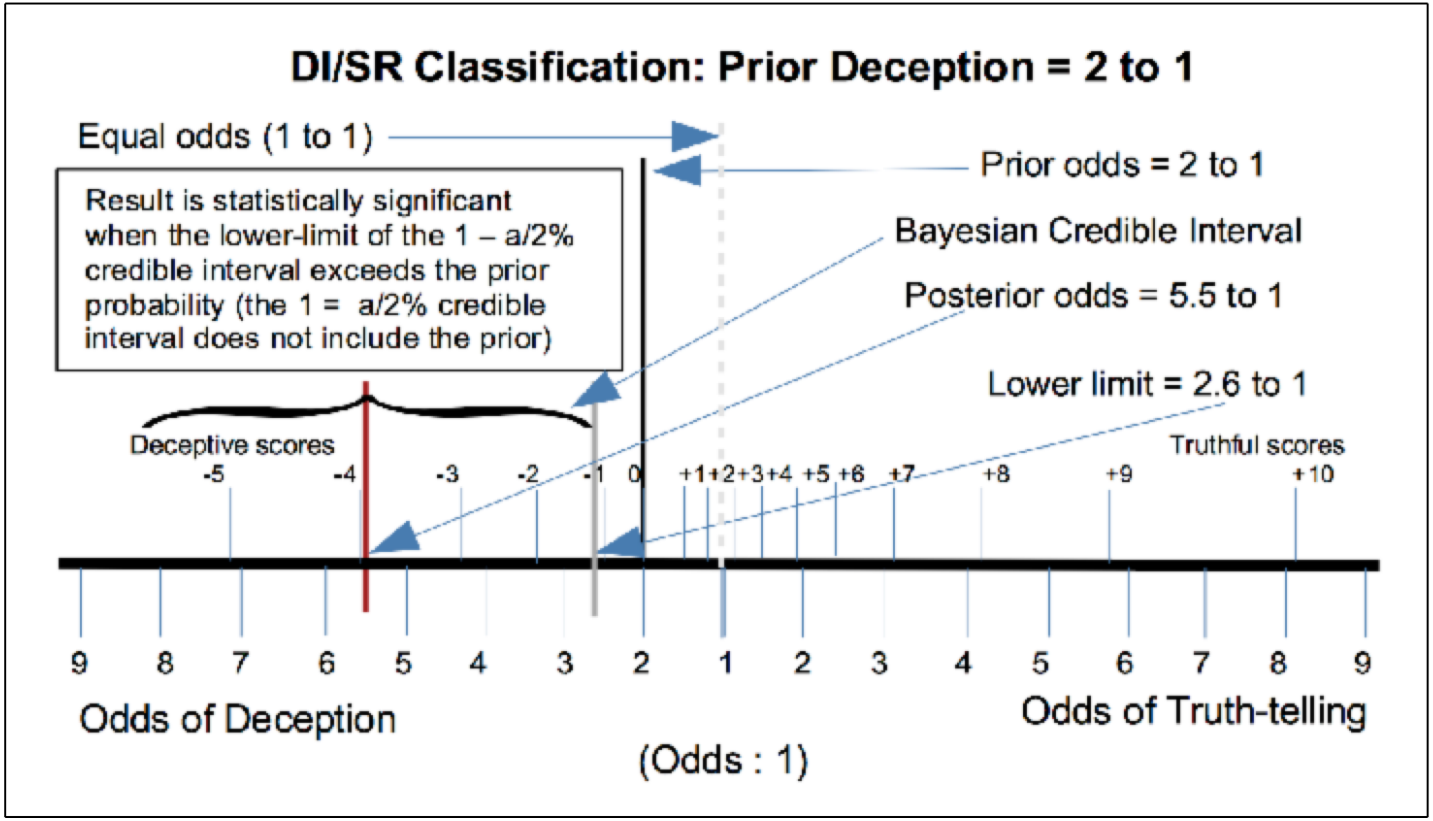
INC/NO (Prior = 2 to 1)

Figure 7. Inconclusive result with prior odds of truth-telling = 2 to 1 (.5) and alpha = .05.



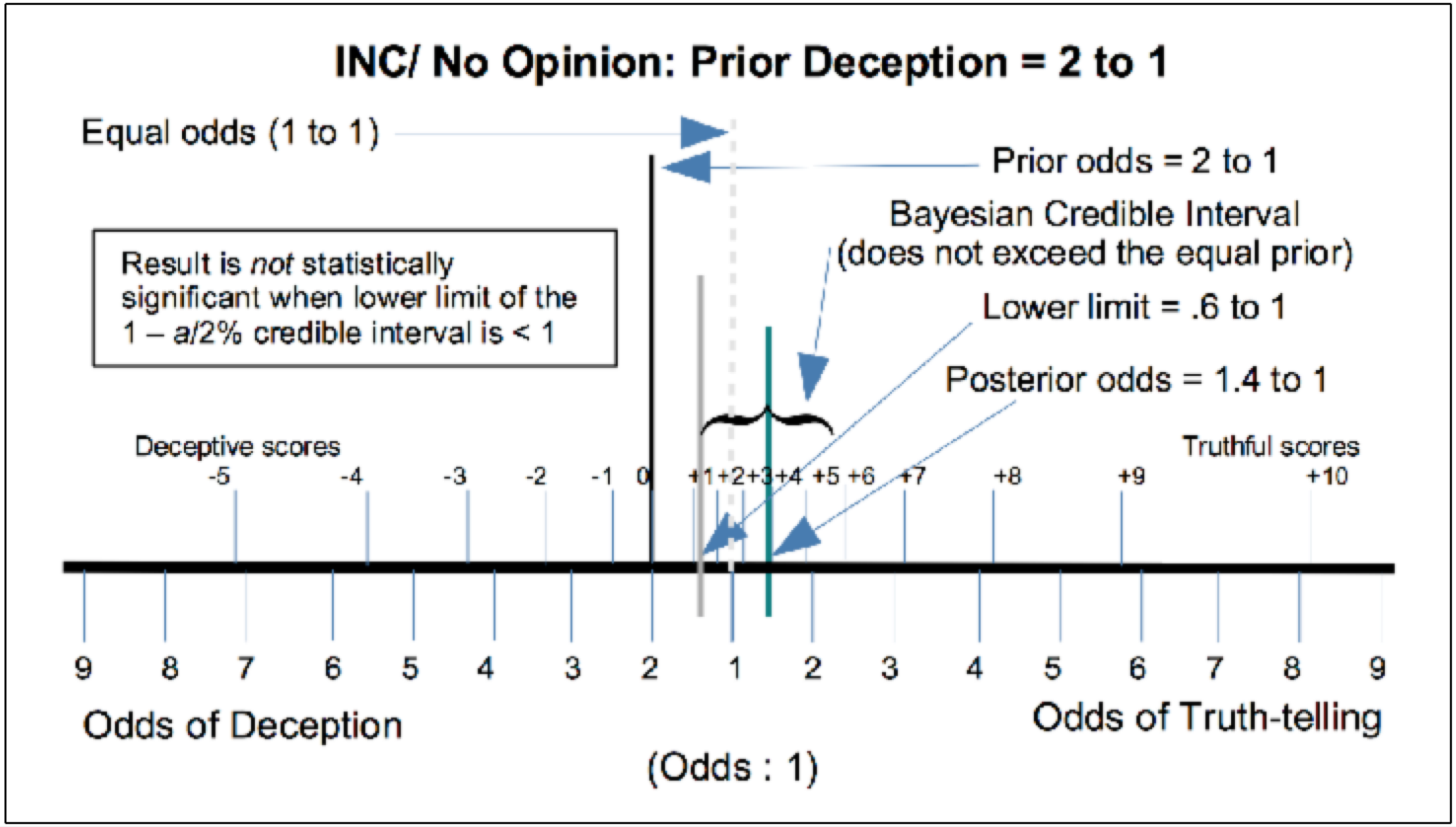
DI/SR (Prior = 2 to 1)

Figure 8. Deceptive test results with prior odds of deception = 2 to 1 and alpha = .05.



INC/NO (Prior = 2 to 1)

Figure 9. Inconclusive result with prior odds of deception = 2 to 1 (.5) and alpha = .05.



Keep it simple

- Use the ESS-Multinomial reference model to determine the cutscores
- Use the cutscores the same way as always
  - A result is statistically significant if the score

Using the Bayesian ESS-M Classifier

- Four parts to any system of test data analysis
  - Scoring features
  - Numerical transformations
  - Statistical reference model
    - Normative data
    - ESS Multinomial Bayesian model
  - Decision rules

ESS-Multinomial Reference Model

- Determine the cutscores
- Calculate the posterior odds (probability)
  - Odds of deception
  - Odds of truth-telling

ESS-Multinomial Reference Table (3RQs)

score	ways	*pmf*	*cdf*	cdfContCor	odds	oddsLL05
-22	360	.0009*	.0025	.0021	483	17.34
-21	370	.0013	.0038	.0031	317.7	16.38
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ESS-Multinomial Reference Tables

- **Score** (grand total)
- **ways** (number of scoresheet permutations that give each score)
- **pmf** (probability for with each score under the null-hypothesis)
- **cdf** (cumlative sum of the pmf)
- **cdfContCor** (continuity corrected cdf – so that the statistical likelihood estimate always exceeds the actual cdf value)
- **odds** (calculated from the cdfContCor)
- **oddsLL05** (lower-limit of the 1-*a*/2% credible interval for the posterior odds of deception or truth)

ESS-Multinomial Reference Tables

- Only 3 columns are useful in field practice
  - Score (grand total)
  - Odds (posterior odds of deception or truth)
    - More informative than the point score
    - More informative than a p-value
  - OddsLL05 (lower limit of the 1-*a*/2% posterior credible interval)
    - Used to determine the cutscores

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ESS-M Multinomial Reference Tables

- Calculated for
  - **Number of RQs**
  - **3 to 5 iterations**
  - **Combined sensor scores**
    - Respiration, EDA, Cardio
    - Respiration, EDA, Cardio, Vasomotor
  - **Prior probability**
    - Prior information is insufficient to make a classification
    - Prior = 1 to 1 is optimal for most purposes
  - **Alpha**
    - *a* = .05 for most purposes
    - Alpha is used to calculate the 1 – *a*/2 credible interval
      - 1 – a/2 CI probability that the posterior probability is different (better) than the prior

ESS-M Multinomial Reference Tables

- Calculated for up to 5 charts
  - Event-specific (diagnostic) exams
    - 2RQs
    - 3RQs
    - 4RQs
  - Multiple-issue (screening) exams
    - 2RQs
    - 3RQs
    - 4RQs

ESS-Multinomial Likelihood Function (3RQs x 3Charts x 3 Sensors)

score	ways	*pmf*	*cdf*	cdfContCor	odds	oddsLL05
-19	90	.0004	.0008	.0006	1712	11.27
-18	100	.0007	.0014	.0011	910.8	11.07
-17	108	.0011	.0025	.0020	503.7	10.73
-16	117	.0018	.0043	.0035	288.9	10.21
-15	124	.0029	.0072	.0058	171.4	9.47
-14	132	.0044	.0116	.0094	105.1	8.52
-13	138	.0064	.0179	.0148	66.37	8.45
-12	145	.0092	.0270	.0227	43.11	7.08
-11	150	.0127	.0394	.0336	28.73	6.28
-10	156	.0169	.0558	.0485	19.62	5.31
-9	160	.0220	.0771	.0681	13.69	4.35
-8	165	.0278	.1037	.0931	9.74	3.6
-7	168	.0341	.1360	.1242	7.05	2.9
-6	172	.0406	.1742	.1617	5.18	2.34
-5	174	.0471	.2181	.2057	3.86	1.87
-4	177	.0531	.2673	.2558	2.91	1.48
-3	178	.0584	.3211	.3115	2.21	1.17
-2	180	.0624	.3786	.3717	1.69	0.91
-1	180	.0649	.4386	.4350	1.3	0.71
0	181	.0658	.5000	.5000	1	0.55
1	180	.0649	.5614	.5650	1.3	0.71
2	180	.0624	.6214	.6283	1.69	0.91
3	178	.0584	.6789	.6885	2.21	1.17
4	177	.0531	.7327	.7442	2.91	1.48
5	174	.0471	.7819	.7943	3.86	1.87
6	172	.0406	.8258	.8383	5.18	2.34
7	168	.0341	.8640	.8758	7.05	2.9
8	165	.0278	.8963	.9069	9.74	3.6
9	160	.0220	.9229	.9319	13.69	4.35
10	156	.0169	.9442	.9515	19.62	5.31
11	150	.0127	.9607	.9664	28.73	6.28
12	145	.0092	.9731	.9773	43.11	7.08

ESS-M Reference Tables

- **Score**
  - Grand total score for all iterations of all RQs
- **Ways**
  - Number of ways (scoresheet permutations) to achieve each score
- **PDF (probability density function)**
  - Proportion of ways to achieve each score / all possible scoresheet permutations
- **CDF (cumulative distribution function)**
  - Running sum of the probabiliti (each value added to the previous sum)
- **contCorCDF (continuity corrected CDF)**
  - Continuity corrected values will always exceed (never equal) the actual CDF
  - Continuity correct for <.5 and >.5
  - No continuity correction for the prior (.5)
- **Odds (posterior odds)**
  - Odds of deception or truth are calculated from the contCorCDF column (p / (1 – p)
- **OddsLL05**
- Lower limit of the 1 – a/2 credible interval for the posterior odds of deception or truth-telling

How to use the ESS-M Tables

How to use the ESS-M Tables

- Alpha determines the upper-limit and lower-limit of the *credible interval*
  - Only the lower-limit offers any interpretable meaning (worst-case scenario)
  - Upper-limit (happy-number) of the credible interval is meaningless/un-interpretable
- Cut-scores are determined by the required alpha level
  - Alpha = .05 for most purposes
- Cut-scores are also determined by the prior odds of deception
  - Prior information is insufficient to conclude deception or truth-telling
  - Prior = 1 to 1 is the optimal prior for most circumstances
  - Published tables are available for the equal prior
- Cut-scores tell us whether or not a result is statistically significant
  - Deception or truth-telling
- ***<u>Cutscores are determined by the lower-limit of the posterior odds</u>***

More on ESS-M Cut-scores

- Cut-scores tell us whether or not a result is statistically significant
  - Deception or truth-telling
- Cut-scores are determined by the prior odds of deception
  - Prior information is insufficient to conclude deception or truth-telling
  - Prior = 1 to 1 is the optimal prior for most circumstances
  - Published tables are available for the equal prior
- Cut-scores are also determined by the required alpha level
  - Alpha = .05 for most purposes
- Alpha determines the upper-limit and lower-limit of the *credible interval*
  - Only the lower-limit offers any interpretable meaning (worst-case scenario)
  - Upper-limit (happy-number) of the credible interval is meaningless/un-interpretable
- ***The lower-limit of the 1-a/2 credible interval determines the cutscore***

How to use the ESS-M Tables (1)

- To get the cut-scores
  - Start with the ***oddsLL05*** column
  - Locate the rows with the *smallest lower-limit odds that exceeds the prior odds*
    - Lower-limit odds for deceptive classification
    - Lower-limit odds for truthful classification
  - Use the corresponding rows in the ***score*** column to determine the cut-scores
    - Cut-score for deception
    - Cut-score for truth-telling

ESS-M Likelihood Function – 3RQs

score	ways	*pmf*	*cdf*	cdfContCor	odds	oddsLL05
-22	360	.0009*	.0025	.0021	483	17.34
-21	370	.0013	.0038	.0031	317.7	16.38
-20	381	.0018	.0056	.0047	212.8	15.18
-18	400	.0035	.0115	.0098	100.6	13.93
-17	408	.0047	.0162	.0139	70.88	12.03
-16	417	.0062	.0223	.0193	50.72	11.12
-15	424	.0080	.0301	.0264	36.84	9.86
-14	432	.0102	.0402	.0355	27.14	8.48
-13	438	.0128	.0526	.0471	20.25	7.15
-12	445	.0157	.0680	.0613	15.31	6.13
-11	450	.0190	.0864	.0787	11.7	5.15
-10	456	.0226	.1081	.0996	9.04	4.27
-9	460	.0264	.1335	.1242	7.05	3.57
-8	465	.0304	.1624	.1526	5.55	2.99
-7	468	.0343	.1950	.1850	4.4	2.48
-6	472	.0382	.2310	.2213	3.52	2.05
-5	474	.0418	.2703	.2613	2.83	1.69
-4	477	.0449	.3125	.3046	2.28	1.4
-3	478	.0476	.3571	.3508	1.85	1.15
-2	480	.0495	.4036	.3992	1.51	0.95
-1	480	.0508	.4515	.4492	1.23	0.77
0	481	.0512	.5000	.5000	1	0.63
1	480	.0508	.5485	.5508	1.23	0.77
2	480	.0495	.5964	.6008	1.51	0.95
3	478	.0476	.6429	.6492	1.85	1.15
4	477	.0449	.6875	.6954	2.28	1.4
5	474	.0418	.7297	.7387	2.83	1.69
6	472	.0382	.7690	.7787	3.52	2.05
7	468	.0343	.8050	.8150	4.4	2.48

Cut-scores: 3RQs

- Deceptive cut-score = **-3**
  - Lower-limit of the 1 – *a*/2 credible interval = 1.15 to 1
- Truthful cut-score = **+3**
  - Lower-limit of the 1 – *a*/2 credible interval = 1.15 to 1

ESS-M Likelihood Function – 3RQs

score	ways	*pmf*	*cdf*	cdfContCor	odds	oddsLL05
-22	360	.0009*	.0025	.0021	483	17.34
-21	370	.0013	.0038	.0031	317.7	16.38
-20	381	.0018	.0056	.0047	212.8	15.18
-18	400	.0035	.0115	.0098	100.6	13.93
-17	408	.0047	.0162	.0139	70.88	12.03
-16	417	.0062	.0223	.0193	50.72	11.12
-15	424	.0080	.0301	.0264	36.84	9.86
-14	432	.0102	.0402	.0355	27.14	8.48
-13	438	.0128	.0526	.0471	20.25	7.15
-12	445	.0157	.0680	.0613	15.31	6.13
-11	450	.0190	.0864	.0787	11.7	5.15
-10	456	.0226	.1081	.0996	9.04	4.27
-9	460	.0264	.1335	.1242	7.05	3.57
-8	465	.0304	.1624	.1526	5.55	2.99
-7	468	.0343	.1950	.1850	4.4	2.48
-6	472	.0382	.2310	.2213	3.52	2.05
-5	474	.0418	.2703	.2613	2.83	1.69
-4	477	.0449	.3125	.3046	2.28	1.4
-3	478	.0476	.3571	.3508	1.85	1.15
-2	480	.0495	.4036	.3992	1.51	0.95
-1	480	.0508	.4515	.4492	1.23	0.77
0	481	.0512	.5000	.5000	1	0.63
1	480	.0508	.5485	.5508	1.23	0.77
2	480	.0495	.5964	.6008	1.51	0.95
3	478	.0476	.6429	.6492	1.85	1.15
4	477	.0449	.6875	.6954	2.28	1.4
5	474	.0418	.7297	.7387	2.83	1.69
6	472	.0382	.7690	.7787	3.52	2.05
7	468	.0343	.8050	.8150	4.4	2.48

How to use the ESS-M Tables (2)

- To get the posterior odds of deception or truth-telling
  - Start with the ***score*** column
  - *Locate the table row that contains the test score*
  - Use the corresponding rows in the ***odds*** column to determine the posterior odds
    - Odds of deception
    - Odds of truth-telling

ESS-M Likelihood Function – Subtotal Scores

score	ways	*pmf*	*cdf*	Cdf ContCor	odds	odds 2RQs	odds 3RQs	odds 4RQs	odds LL05	odds2RQLL05	odds3RQLL05	odds4RQLL05
-14	16	.0005*	.0007	.0005	1970	44.38	12.54	6.66	6.11	4.19	2.85	2.1
-13	20	.0011	.0018	.0013	778.5	27.9	9.2	5.28	6.01	4	2.46	1.8
-12	25	.0022	.0040	.0029	339.5	18.43	6.98	4.29	5.82	3.56	2.17	1.55
-11	30	.0042	.0082	.0062	161.1	12.69	5.44	3.56	5.46	2.87	1.84	1.35
-10	36	.0074	.0156	.0120	82.2	9.07	4.35	3.01	4.92	2.44	1.57	1.18
-9	40	.0122	.0275	.0219	44.7	6.69	3.55	2.59	4.2	2.11	1.34	1.05
-8	45	.0188	.0458	.0375	25.68	5.07	2.95	2.25	3.86	1.74	1.17	0.93
-7	48	.0272	.0719	.0607	15.48	3.94	2.49	1.98	3.23	1.47	1.02	0.83
-6	52	.0374	.1072	.0933	9.72	3.12	2.13	1.77	2.56	1.22	0.89	0.75
-5	54	.0487	.1524	.1367	6.32	2.51	1.85	1.59	2.02	1.02	0.78	0.68
-4	57	.0602	.2075	.1914	4.23	2.06	1.62	1.43	1.53	0.86	0.69	0.62
-3	58	.0710	.2717	.2571	2.89	1.7	1.42	1.3	1.15	0.72	0.61	0.56
-2	60	.0798	.3434	.3322	2.01	1.42	1.26	1.19	0.84	0.61	0.54	0.51
-1	60	.0855	.4203	.4143	1.41	1.19	1.12	1.09	0.61	0.51	0.48	0.47
0	61	.0875	.5000	.5000	1	1	1	1	0.43	0.43	0.43	0.43
1	60	.0855	.5797	.5857	1.41	2	2.83	4	0.61	0.84	1.13	1.49
2	60	.0798	.6566	.6678	2.01	4.04	8.12	16.32	0.84	1.47	2.35	3.33
3	58	.0710	.7283	.7429	2.89	8.35	24.13	69.71	1.15	2.4	3.75	4.75
4	57	.0602	.7925	.8086	4.23	17.85	75.4	318.5	1.53	3.5	4.83	5.79
5	54	.0487	.8476	.8633	6.32	39.91	252.2	1593	2.02	4.05	5.7	6.1
6	52	.0374	.8928	.9067	9.72	94.48	918.4	8927	2.56	5.05	6.04	6.16
7	48	.0272	.9281	.9393	15.48	239.6	3710	57430	3.23	5.68	6.14	6.17

ESS-M Likelihood Function – Subtotal Scores

- Odds
  - OddsLL05
- Odds2RQ
  - Odds2RQLL05
- Odds3RQ
  - Odds3RQLL05
- Odds4RQ
  - Odds4RQLL05

ESS-M Likelihood Function – Subtotal Scores

score	ways	*pmf*	*cdf*	Cdf ContCor	odds	odds 2RQs	odds 3RQs	odds 4RQs	odds LL05	odds2RQLL05	odds3RQLL05	odds4RQLL05
-14	16	.0005*	.0007	.0005	1970	44.38	12.54	6.66	6.11	4.19	2.85	2.1
-13	20	.0011	.0018	.0013	778.5	27.9	9.2	5.28	6.01	4	2.46	1.8
-12	25	.0022	.0040	.0029	339.5	18.43	6.98	4.29	5.82	3.56	2.17	1.55
-11	30	.0042	.0082	.0062	161.1	12.69	5.44	3.56	5.46	2.87	1.84	1.35
-10	36	.0074	.0156	.0120	82.2	9.07	4.35	3.01	4.92	2.44	1.57	1.18
-9	40	.0122	.0275	.0219	44.7	6.69	3.55	2.59	4.2	2.11	1.34	1.05
-8	45	.0188	.0458	.0375	25.68	5.07	2.95	2.25	3.86	1.74	1.17	0.93
-7	48	.0272	.0719	.0607	15.48	3.94	2.49	1.98	3.23	1.47	1.02	0.83
-6	52	.0374	.1072	.0933	9.72	3.12	2.13	1.77	2.56	1.22	0.89	0.75
-5	54	.0487	.1524	.1367	6.32	2.51	1.85	1.59	2.02	1.02	0.78	0.68
-4	57	.0602	.2075	.1914	4.23	2.06	1.62	1.43	1.53	0.86	0.69	0.62
-3	58	.0710	.2717	.2571	2.89	1.7	1.42	1.3	1.15	0.72	0.61	0.56
-2	60	.0798	.3434	.3322	2.01	1.42	1.26	1.19	0.84	0.61	0.54	0.51
-1	60	.0855	.4203	.4143	1.41	1.19	1.12	1.09	0.61	0.51	0.48	0.47
0	61	.0875	.5000	.5000	1	1	1	1	0.43	0.43	0.43	0.43
1	60	.0855	.5797	.5857	1.41	2	2.83	4	0.61	0.84	1.13	1.49
2	60	.0798	.6566	.6678	2.01	4.04	8.12	16.32	0.84	1.47	2.35	3.33
3	58	.0710	.7283	.7429	2.89	8.35	24.13	69.71	1.15	2.4	3.75	4.75
4	57	.0602	.7925	.8086	4.23	17.85	75.4	318.5	1.53	3.5	4.83	5.79
5	54	.0487	.8476	.8633	6.32	39.91	252.2	1593	2.02	4.05	5.7	6.1
6	52	.0374	.8928	.9067	9.72	94.48	918.4	8927	2.56	5.05	6.04	6.16
7	48	.0272	.9281	.9393	15.48	239.6	3710	57430	3.23	5.68	6.14	6.17
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

How to use the Multinomial Subtotal Tables (1)

- To get the cut-scores
  - Determine the number of RQs
  - Select from ***odds2RQLL05***, ***odds3RQLL05***, ***or odds4RQLL05***
  - Locate the rows with the *smallest lower-limit odds that exceeds the prior odds*
    - Lower-limit odds for deceptive classification
    - Lower-limit odds for truthful classification
  - Use the corresponding rows in the ***score*** column to determine the cut-scores
    - Cut-score for deception
    - Cut-score for truth-telling

ESS-M Likelihood Function – Subtotal Scores

score	ways	*pmf*	*cdf*	Cdf ContCor	odds	odds 2RQs	odds 3RQs	odds 4RQs	odds LL05	odds2RQLL05	odds3RQLL05	odds4RQLL05
-14	16	.0005*	.0007	.0005	1970	44.38	12.54	6.66	6.11	4.19	2.85	2.1
-13	20	.0011	.0018	.0013	778.5	27.9	9.2	5.28	6.01	4	2.46	1.8
-12	25	.0022	.0040	.0029	339.5	18.43	6.98	4.29	5.82	3.56	2.17	1.55
-11	30	.0042	.0082	.0062	161.1	12.69	5.44	3.56	5.46	2.87	1.84	1.35
-10	36	.0074	.0156	.0120	82.2	9.07	4.35	3.01	4.92	2.44	1.57	1.18
-9	40	.0122	.0275	.0219	44.7	6.69	3.55	2.59	4.2	2.11	1.34	1.05
-8	45	.0188	.0458	.0375	25.68	5.07	2.95	2.25	3.86	1.74	1.17	0.93
-7	48	.0272	.0719	.0607	15.48	3.94	2.49	1.98	3.23	1.47	1.02	0.83
-6	52	.0374	.1072	.0933	9.72	3.12	2.13	1.77	2.56	1.22	0.89	0.75
-5	54	.0487	.1524	.1367	6.32	2.51	1.85	1.59	2.02	1.02	0.78	0.68
-4	57	.0602	.2075	.1914	4.23	2.06	1.62	1.43	1.53	0.86	0.69	0.62
-3	58	.0710	.2717	.2571	2.89	1.7	1.42	1.3	1.15	0.72	0.61	0.56
-2	60	.0798	.3434	.3322	2.01	1.42	1.26	1.19	0.84	0.61	0.54	0.51
-1	60	.0855	.4203	.4143	1.41	1.19	1.12	1.09	0.61	0.51	0.48	0.47
0	61	.0875	.5000	.5000	1	1	1	1	0.43	0.43	0.43	0.43
1	60	.0855	.5797	.5857	1.41	2	2.83	4	0.61	0.84	1.13	1.49
2	60	.0798	.6566	.6678	2.01	4.04	8.12	16.32	0.84	1.47	2.35	3.33
3	58	.0710	.7283	.7429	2.89	8.35	24.13	69.71	1.15	2.4	3.75	4.75
4	57	.0602	.7925	.8086	4.23	17.85	75.4	318.5	1.53	3.5	4.83	5.79
5	54	.0487	.8476	.8633	6.32	39.91	252.2	1593	2.02	4.05	5.7	6.1
6	52	.0374	.8928	.9067	9.72	94.48	918.4	8927	2.56	5.05	6.04	6.16
7	48	.0272	.9281	.9393	15.48	239.6	3710	57430	3.23	5.68	6.14	6.17

When to use the statistical correction

- Event-specific (diagnostic) exams
  - No statistical correction for grand total scores
  - Use the statistical correction for deceptive subtotals with the TSR
  - Truthful subtotal scores are not used with the TSR
- Multiple-issue (screening) exams
  - No statistical correction for deceptive subtotals
    - Common in screening to avoid loss of test sensitivity
  - Use statistical correction for truthful subtotals
    - Reduces inconclusive results for innocent persons
    - Use of the lowest subtotal means that passing the test requires passing all RQs

ESS-M Likelihood Function – Subtotal Scores

score	ways	*pmf*	*cdf*	Cdf ContCor	odds	odds 2RQs	odds 3RQs	odds 4RQs	odds LL05	odds2RQLL05	odds3RQLL05	odds4RQLL05
-14	16	.0005*	.0007	.0005	1970	44.38	12.54	6.66	6.11	4.19	2.85	2.1
-13	20	.0011	.0018	.0013	778.5	27.9	9.2	5.28	6.01	4	2.46	1.8
-12	25	.0022	.0040	.0029	339.5	18.43	6.98	4.29	5.82	3.56	2.17	1.55
-11	30	.0042	.0082	.0062	161.1	12.69	5.44	3.56	5.46	2.87	1.84	1.35
-10	36	.0074	.0156	.0120	82.2	9.07	4.35	3.01	4.92	2.44	1.57	1.18
-9	40	.0122	.0275	.0219	44.7	6.69	3.55	2.59	4.2	2.11	1.34	1.05
-8	45	.0188	.0458	.0375	25.68	5.07	2.95	2.25	3.86	1.74	1.17	0.93
-7	48	.0272	.0719	.0607	15.48	3.94	2.49	1.98	3.23	1.47	1.02	0.83
-6	52	.0374	.1072	.0933	9.72	3.12	2.13	1.77	2.56	1.22	0.89	0.75
-5	54	.0487	.1524	.1367	6.32	2.51	1.85	1.59	2.02	1.02	0.78	0.68
-4	57	.0602	.2075	.1914	4.23	2.06	1.62	1.43	1.53	0.86	0.69	0.62
-3	58	.0710	.2717	.2571	2.89	1.7	1.42	1.3	1.15	0.72	0.61	0.56
-2	60	.0798	.3434	.3322	2.01	1.42	1.26	1.19	0.84	0.61	0.54	0.51
-1	60	.0855	.4203	.4143	1.41	1.19	1.12	1.09	0.61	0.51	0.48	0.47
0	61	.0875	.5000	.5000	1	1	1	1	0.43	0.43	0.43	0.43
1	60	.0855	.5797	.5857	1.41	2	2.83	4	0.61	0.84	1.13	1.49
2	60	.0798	.6566	.6678	2.01	4.04	8.12	16.32	0.84	1.47	2.35	3.33
3	58	.0710	.7283	.7429	2.89	8.35	24.13	69.71	1.15	2.4	3.75	4.75
4	57	.0602	.7925	.8086	4.23	17.85	75.4	318.5	1.53	3.5	4.83	5.79
5	54	.0487	.8476	.8633	6.32	39.91	252.2	1593	2.02	4.05	5.7	6.1
6	52	.0374	.8928	.9067	9.72	94.48	918.4	8927	2.56	5.05	6.04	6.16
7	48	.0272	.9281	.9393	15.48	239.6	3710	57430	3.23	5.68	6.14	6.17

Cut-scores: Sub-total Scores - Screening

- Deceptive cut-score = **-3**
  - Lower-limit of the 1 – *a*/2 credible interval = 1.15 to 1
- Truthful cut-score
  - 2RQs = **+2**
    - Lower-limit of the 1 – a/2 credible interval = 1.47 to 1
  - 3RQs = **+1**
    - Lower-limit of the 1 – a/2 credible interval = 1.13 to 1
  - 4RQs = **+1**
    - Lower-limit of the 1 – a/2 credible interval = 1.49 to 1

Cut-scores: Sub-total Scores - Diagnostic

- Truthful cut-score is not used
  - Subtotal scores are not used for truthful classifications of diagnostic exams
- Deceptive cut-scores
  - 2RQs = **-5**
    - Lower-limit of the 1 – a/2 credible interval = 1.02 to 1
  - 3RQs = **-7**
    - Lower-limit of the 1 – a/2 credible interval = 1.02 to 1
  - 4RQs = **-9**
    - Lower-limit of the 1 – a/2 credible interval = 1.05 to 1

ESS-M Cutscores

- Single issue exams

	2 RQs	3 RQs	4RQs
Respiration, EDA, Cardio	+3 / -3 (-5)	+3 / -3 (-7)	+3 / -3 (-9)
Respiration, EDA, Cardio, Vasomotor	+3 / -3 (-5)	+3 / -3 (-7)	+3 / -3 (-9)

- Multiple issue exams

	2 RQs	3 RQs	4RQs
Respiration, EDA, Cardio	+2 / -3	+1 / -3	+1 / -3
Respiration, EDA, Cardio, Vasomotor	+2 / -3	+1 / -3	+1 / -3

How to use the Multinomial Subtotal Tables (2)

- To get the posterior odds of deception or truth-telling
  - Start with the ***score*** column
  - *Locate the table row that contains the test score*
  - Determine the number of RQs
    - Use the corresponding rows in the ***odds2RQ***, ***odds3RQ****, or* ***odds4RQ*** column to determine the posterior odds
      - Odds of deception
      - Odds of truth-telling

Examples

Example 1: 3 RQ Diagnostic Exam

R1 = -4
R2 = -5
R3 = -3
Grand total = -12

ESS-M Likelihood Function – 3RQs

score	ways	*pmf*	*cdf*	cdfContCor	odds	oddsLL05
-22	360	.0009*	.0025	.0021	483	17.34
-21	370	.0013	.0038	.0031	317.7	16.38
-20	381	.0018	.0056	.0047	212.8	15.18
-18	400	.0035	.0115	.0098	100.6	13.93
-17	408	.0047	.0162	.0139	70.88	12.03
-16	417	.0062	.0223	.0193	50.72	11.12
-15	424	.0080	.0301	.0264	36.84	9.86
-14	432	.0102	.0402	.0355	27.14	8.48
-13	438	.0128	.0526	.0471	20.25	7.15
-12	445	.0157	.0680	.0613	15.31	6.13
-11	450	.0190	.0864	.0787	11.7	5.15
-10	456	.0226	.1081	.0996	9.04	4.27
-9	460	.0264	.1335	.1242	7.05	3.57
-8	465	.0304	.1624	.1526	5.55	2.99
-7	468	.0343	.1950	.1850	4.4	2.48
-6	472	.0382	.2310	.2213	3.52	2.05
-5	474	.0418	.2703	.2613	2.83	1.69
-4	477	.0449	.3125	.3046	2.28	1.4
-3	478	.0476	.3571	.3508	1.85	1.15
-2	480	.0495	.4036	.3992	1.51	0.95
-1	480	.0508	.4515	.4492	1.23	0.77
0	481	.0512	.5000	.5000	1	0.63
1	480	.0508	.5485	.5508	1.23	0.77
2	480	.0495	.5964	.6008	1.51	0.95
3	478	.0476	.6429	.6492	1.85	1.15
4	477	.0449	.6875	.6954	2.28	1.4
5	474	.0418	.7297	.7387	2.83	1.69
6	472	.0382	.7690	.7787	3.52	2.05
7	468	.0343	.8050	.8150	4.4	2.48

Example 1: 3 RQ Diagnostic Exam

Grand total = -12
Posterior odds of deception = 15 to 1
Posterior probability = .94

Example 2: 3 RQ Diagnostic Exam

R1 = +2
R2 = +2
R3 = +1
Grand total = +5

ESS-M Likelihood Function – 3RQs

score	ways	*pmf*	*cdf*	cdfContCor	odds	oddsLL05
-22	360	.0009*	.0025	.0021	483	17.34
-21	370	.0013	.0038	.0031	317.7	16.38
-20	381	.0018	.0056	.0047	212.8	15.18
-18	400	.0035	.0115	.0098	100.6	13.93
-17	408	.0047	.0162	.0139	70.88	12.03
-16	417	.0062	.0223	.0193	50.72	11.12
-15	424	.0080	.0301	.0264	36.84	9.86
-14	432	.0102	.0402	.0355	27.14	8.48
-13	438	.0128	.0526	.0471	20.25	7.15
-12	445	.0157	.0680	.0613	15.31	6.13
-11	450	.0190	.0864	.0787	11.7	5.15
-10	456	.0226	.1081	.0996	9.04	4.27
-9	460	.0264	.1335	.1242	7.05	3.57
-8	465	.0304	.1624	.1526	5.55	2.99
-7	468	.0343	.1950	.1850	4.4	2.48
-6	472	.0382	.2310	.2213	3.52	2.05
-5	474	.0418	.2703	.2613	2.83	1.69
-4	477	.0449	.3125	.3046	2.28	1.4
-3	478	.0476	.3571	.3508	1.85	1.15
-2	480	.0495	.4036	.3992	1.51	0.95
-1	480	.0508	.4515	.4492	1.23	0.77
0	481	.0512	.5000	.5000	1	0.63
1	480	.0508	.5485	.5508	1.23	0.77
2	480	.0495	.5964	.6008	1.51	0.95
3	478	.0476	.6429	.6492	1.85	1.15
4	477	.0449	.6875	.6954	2.28	1.4
5	474	.0418	.7297	.7387	2.83	1.69
6	472	.0382	.7690	.7787	3.52	2.05
7	468	.0343	.8050	.8150	4.4	2.48

Example 2: 3 RQ Diagnostic Exam

Grand total = +5
Posterior odds of deception = 2.8 to 1
Posterior probability = .74

Example 3: Subtotal Scores (multi-issue)

Always use the lowest subtotal score

R1 = +2

R2 = +3

R3 = -4 ← lowest subtotal score

R4 = +1

No grand total score for the SSR

ESS-M Likelihood Function – Subtotal Scores

score	ways	*pmf*	*cdf*	Cdf ContCor	odds	odds 2RQs	odds 3RQs	odds 4RQs	odds LL05	odds2RQLL05	odds3RQLL05	odds4RQLL05
-14	16	.0005*	.0007	.0005	1970	44.38	12.54	6.66	6.11	4.19	2.85	2.1
-13	20	.0011	.0018	.0013	778.5	27.9	9.2	5.28	6.01	4	2.46	1.8
-12	25	.0022	.0040	.0029	339.5	18.43	6.98	4.29	5.82	3.56	2.17	1.55
-11	30	.0042	.0082	.0062	161.1	12.69	5.44	3.56	5.46	2.87	1.84	1.35
-10	36	.0074	.0156	.0120	82.2	9.07	4.35	3.01	4.92	2.44	1.57	1.18
-9	40	.0122	.0275	.0219	44.7	6.69	3.55	2.59	4.2	2.11	1.34	1.05
-8	45	.0188	.0458	.0375	25.68	5.07	2.95	2.25	3.86	1.74	1.17	0.93
-7	48	.0272	.0719	.0607	15.48	3.94	2.49	1.98	3.23	1.47	1.02	0.83
-6	52	.0374	.1072	.0933	9.72	3.12	2.13	1.77	2.56	1.22	0.89	0.75
-5	54	.0487	.1524	.1367	6.32	2.51	1.85	1.59	2.02	1.02	0.78	0.68
-4	57	.0602	.2075	.1914	4.23	2.06	1.62	1.43	1.53	0.86	0.69	0.62
-3	58	.0710	.2717	.2571	2.89	1.7	1.42	1.3	1.15	0.72	0.61	0.56
-2	60	.0798	.3434	.3322	2.01	1.42	1.26	1.19	0.84	0.61	0.54	0.51
-1	60	.0855	.4203	.4143	1.41	1.19	1.12	1.09	0.61	0.51	0.48	0.47
0	61	.0875	.5000	.5000	1	1	1	1	0.43	0.43	0.43	0.43
1	60	.0855	.5797	.5857	1.41	2	2.83	4	0.61	0.84	1.13	1.49
2	60	.0798	.6566	.6678	2.01	4.04	8.12	16.32	0.84	1.47	2.35	3.33
3	58	.0710	.7283	.7429	2.89	8.35	24.13	69.71	1.15	2.4	3.75	4.75
4	57	.0602	.7925	.8086	4.23	17.85	75.4	318.5	1.53	3.5	4.83	5.79
5	54	.0487	.8476	.8633	6.32	39.91	252.2	1593	2.02	4.05	5.7	6.1
6	52	.0374	.8928	.9067	9.72	94.48	918.4	8927	2.56	5.05	6.04	6.16
7	48	.0272	.9281	.9393	15.48	239.6	3710	57430	3.23	5.68	6.14	6.17
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Example 3: Subtotal Score

Lowest subtotal score = -4
Posterior odds of deception = 4.2 to 1
Posterior probability = .81

Example 4: Subtotal Scores (multi-issue)

Always use the lowest subtotal score
R1 = +1 ← lowest subtotal score
R2 = +2
R3 = +3
R4 = +4
No grand total score for the SSR

ESS-M Likelihood Function – Subtotal Scores

score	ways	*pmf*	*cdf*	Cdf ContCor	odds	odds 2RQs	odds 3RQs	odds 4RQs	odds LL05	odds2RQLL05	odds3RQLL05	odds4RQLL05
-14	16	.0005*	.0007	.0005	1970	44.38	12.54	6.66	6.11	4.19	2.85	2.1
-13	20	.0011	.0018	.0013	778.5	27.9	9.2	5.28	6.01	4	2.46	1.8
-12	25	.0022	.0040	.0029	339.5	18.43	6.98	4.29	5.82	3.56	2.17	1.55
-11	30	.0042	.0082	.0062	161.1	12.69	5.44	3.56	5.46	2.87	1.84	1.35
-10	36	.0074	.0156	.0120	82.2	9.07	4.35	3.01	4.92	2.44	1.57	1.18
-9	40	.0122	.0275	.0219	44.7	6.69	3.55	2.59	4.2	2.11	1.34	1.05
-8	45	.0188	.0458	.0375	25.68	5.07	2.95	2.25	3.86	1.74	1.17	0.93
-7	48	.0272	.0719	.0607	15.48	3.94	2.49	1.98	3.23	1.47	1.02	0.83
-6	52	.0374	.1072	.0933	9.72	3.12	2.13	1.77	2.56	1.22	0.89	0.75
-5	54	.0487	.1524	.1367	6.32	2.51	1.85	1.59	2.02	1.02	0.78	0.68
-4	57	.0602	.2075	.1914	4.23	2.06	1.62	1.43	1.53	0.86	0.69	0.62
-3	58	.0710	.2717	.2571	2.89	1.7	1.42	1.3	1.15	0.72	0.61	0.56
-2	60	.0798	.3434	.3322	2.01	1.42	1.26	1.19	0.84	0.61	0.54	0.51
-1	60	.0855	.4203	.4143	1.41	1.19	1.12	1.09	0.61	0.51	0.48	0.47
0	61	.0875	.5000	.5000	1	1	1	1	0.43	0.43	0.43	0.43
1	60	.0855	.5797	.5857	1.41	2	2.83	4	0.61	0.84	1.13	1.49
2	60	.0798	.6566	.6678	2.01	4.04	8.12	16.32	0.84	1.47	2.35	3.33
3	58	.0710	.7283	.7429	2.89	8.35	24.13	69.71	1.15	2.4	3.75	4.75
4	57	.0602	.7925	.8086	4.23	17.85	75.4	318.5	1.53	3.5	4.83	5.79
5	54	.0487	.8476	.8633	6.32	39.91	252.2	1593	2.02	4.05	5.7	6.1
6	52	.0374	.8928	.9067	9.72	94.48	918.4	8927	2.56	5.05	6.04	6.16
7	48	.0272	.9281	.9393	15.48	239.6	3710	57430	3.23	5.68	6.14	6.17
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

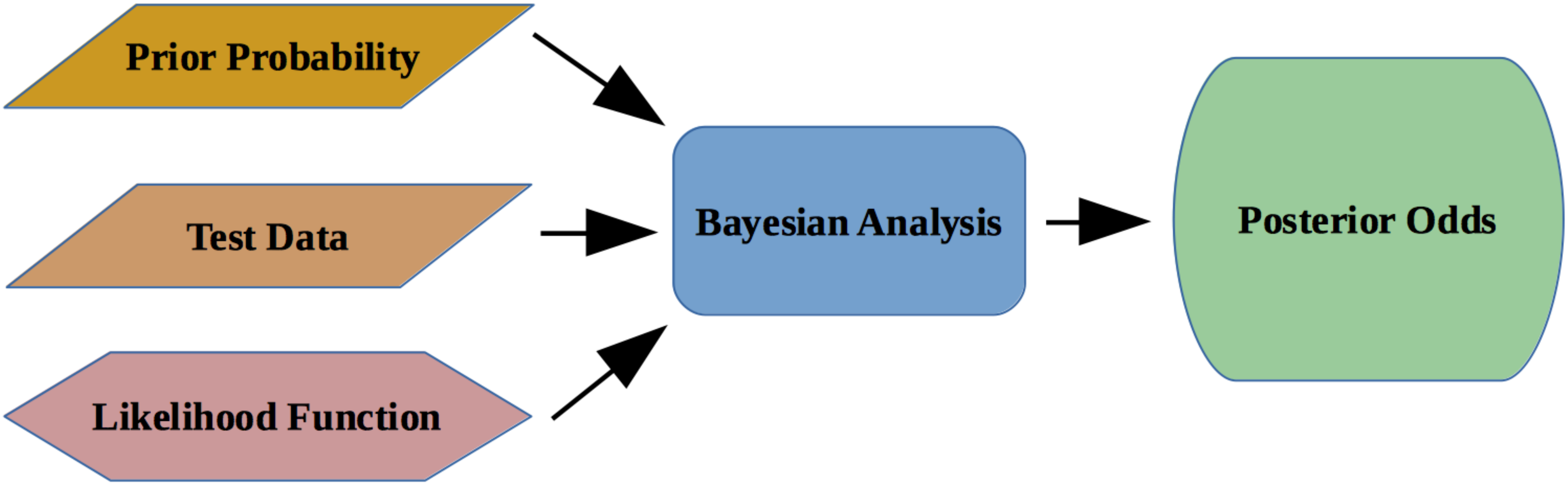
Example 4: Subtotal Score

Lowest subtotal score = +1
Posterior odds of deception = 1.5 to 1
Posterior probability = .60

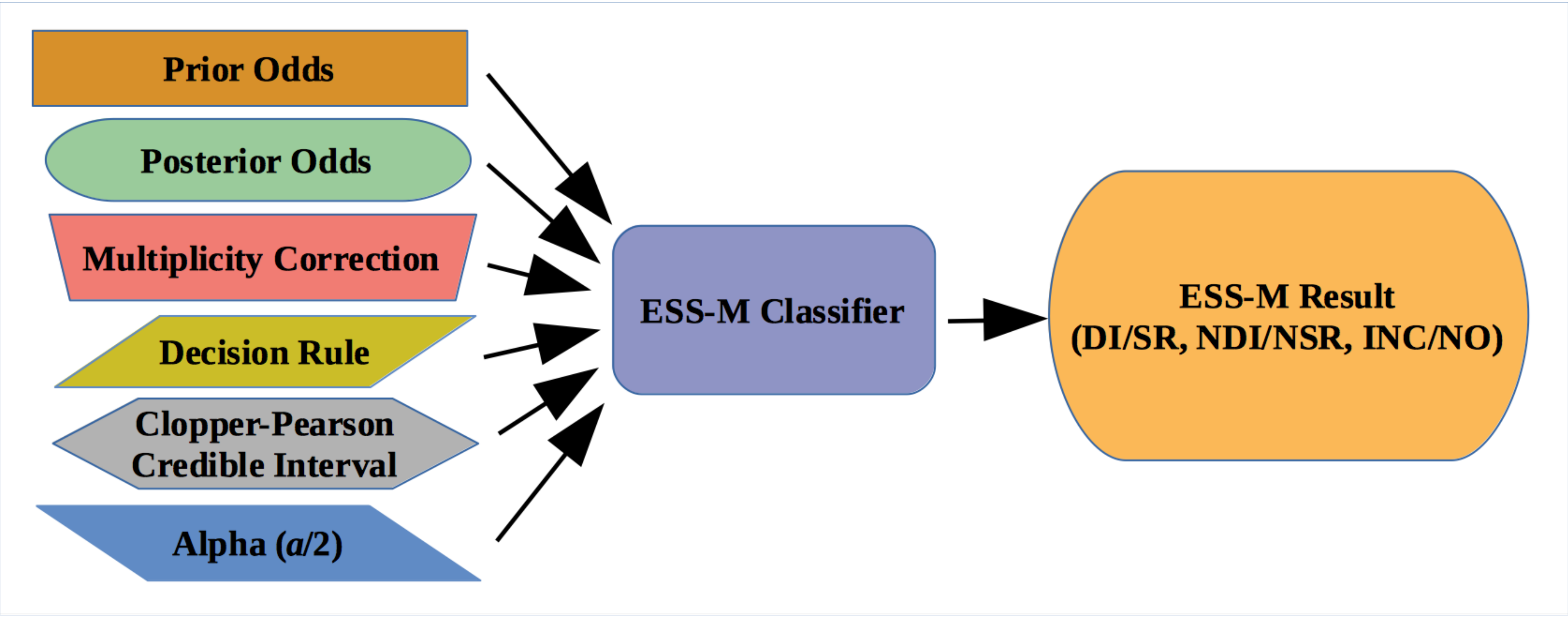
Bayesian analytics?

- Provides a more intuitive statistical estimate of the effect size of practical interest
  - Deception
  - Truth-telling
- Bayesian posterior odds (posterior probabilities) are more intuitive and less vulnerable than frequentist p-values
  - Less vulnerable to misunderstanding
  - Less vulnerable to abuse
  - Less vulnerable to overestimation

Bayesian Analysis



Bayesian ESS-Multinomial Classifier



Bayes – Vocabulary Primer

- Bayesian inference
- Bayes Theorem
- Probability (Bayesian probability)
- Prior probability (prior probability distribution or *a priori*)
- Likelihood function
- Posterior probability (*a posteriori*)
- Odds
- Bayes Factor
- Credible interval
- Naive-Bayes
- Objective Bayesian Analysis
- Subjective Bayesian Analysis

The End.

raymond.nelson@gmail.com 1-917-293-3208